- This exam is worth 150 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on both sides.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/050724 (16 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) Given any three nonzero vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$, if $\mathbf{u} \times \mathbf{v} \times \mathbf{w} = \mathbf{0}$, then the vectors must always all lie in the same plane.

   (b) The torsion, $\tau = \dfrac{[\mathbf{r}'(t) \times \mathbf{r}''(t)] \cdot \mathbf{r}'''(t)}{||\mathbf{r}'(t) \times \mathbf{r}''(t)||^2}$, of the curve $\mathbf{r}(t) = \langle \sin t, 2, \cos t \rangle$ is $\tau = 2$.

   (c) The line with symmetric equations $x = -0.5y = z$ never intersects the plane $x + y + z = 1$

   (d) $\dfrac{1}{4}x^2 - 2x + y^2 - 10y - z^2 + 39 = 0$ is a hyperboloid of two sheets.

   (e) For any vectors $\mathbf{A}, \mathbf{B}$, the operation $\nabla \times [\nabla \times (\mathbf{A} \times \mathbf{B})]$ is well-defined.

   (f) The direction of motion of a particle moving on the path $\mathbf{r}(t) = (1 - 3e^t)\,\mathbf{i} + (2 - 2e^t)\,\mathbf{j} + (3 - e^t)\,\mathbf{k}, t \in \mathbb{R}$, is always changing (the normal component of the acceleration is never 0) but its speed is constant (its tangential acceleration is 0).

   (g) If $g(x, y) \to 1$ when $(x, y) \to (0, 0)$ along the y-axis and $g(x, y) \to 1$ when $(x, y) \to (0, 0)$ along the x-axis, and $g(0, 0) = 1$, then $g(x, y)$ must be continuous at $(0, 0)$.

   (h) The linear Taylor polynomial of $f(x, y) = e^{-x^4 - y^4}$ centered at $(1, 1)$ is $T_1(x, y) = e^{-2}(9 - 4x - 4y)$.

2. [2350/050724 (30 pts)] Consider the triangular region, $\mathcal{D}$, with boundary, $\partial\mathcal{D}$, given by $x = 1, y = 0, y = x$, and the vector field $\mathbf{F} = 3(x - 1)^2 y\,\mathbf{i} + (x - 1)y^3\,\mathbf{j}$.

   (a) (15 pts) Without using any Calculus 3 theorems, directly compute the circulation of $\mathbf{F}$ along $\partial\mathcal{D}$ with *clockwise* orientation.

   (b) (15 pts) Use Green's Theorem to compute the outward flux of $\mathbf{F}$ through $\partial\mathcal{D}$.

3. [2350/050724 (30 pts)] Consider the function $f(x, y) = x^4 + y^4 - 2x^2 - 4y$

   (a) (11 pts) Suppose you are standing on the surface with $x = 2, y = -1$.

      i. (2 pts) What is the distance to the xy-plane from your position?

      ii. (2 pts) Is the xy-plane above or below you?

      iii. (7 pts) Find a unit vector in the xy-plane showing the direction you would need to move to follow the level curve that passes through your x, y coordinates.

   (b) (7 pts) Suppose you are walking on the surface along a path whose projection on the xy-plane is $\mathbf{r}(t) = \left(4t - \frac{3}{2}\right)\mathbf{i} + 2t\,\mathbf{j}$. Find the rate of change with respect to time of your z-coordinate when your path passes through the point $(x, y, z) = \left(-\frac{1}{2}, \frac{1}{2}, -\frac{19}{8}\right)$.

   (c) (12 pts) Find and classify all critical points of the function.

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4. [2350/050724 (20 pts)] A butterfly of the species *Infinitus spectacularis* has been tagged with a radio receiver that measures the amount of work it does when flying around the first octant. The vector field in which the butterfly is flying is

$$\mathbf{F} = \left[ye^{xy}\ln(yz) - z\sin(xz)\right]\mathbf{i} + \left[\frac{e^{xy}}{y} + xe^{xy}\ln(yz)\right]\mathbf{j} + \left[\frac{e^{xy}}{z} - x\sin(xz) + 3z^2\right]\mathbf{k}$$

Biologist records over time indicate that the work done by the butterfly for every closed path it flies is always zero. How much work is done by the butterfly if it flies along the straight line path from $(1,1,1)$ to $\left(\frac{1}{2},4,2\right)$?

5. [2350/050724 (34 pts)] Let $\mathcal{W}$ be the solid bounded by the planes $x=0, y=0, z=2$ and the portion of $2z = x^2+y^2$ above the fourth quadrant, and let $\partial\mathcal{W}$ denote its boundary. We will be considering the vector field $\mathbf{E} = \left\langle x^2y, xy^2, xy(z-2)\right\rangle$. The identity $\sin 2x = 2\sin x\cos x$ might be helpful.

   (a) (4 pts) Briefly explain why computing the flux of $\mathbf{E}$ through $\partial\mathcal{W}$ requires evaluating only a single nontrivial integral.

   (b) (15 pts) Find the outward flux of $\mathbf{E}$ through $\partial\mathcal{W}$ by direct calculation.

   (c) (15 pts) Use an important Calculus 3 theorem to compute the outward flux of $\mathbf{E}$ through $\partial\mathcal{W}$ another way.

6. [2350/050724 (20 pts)] Use Stokes theorem to evaluate $\displaystyle\int_{\partial\mathcal{S}} x\,\mathrm{d}x + (x-2yz)\,\mathrm{d}y + \left(x^2+z^4\right)\mathrm{d}z$. $\partial\mathcal{S}$ and its orientation is shown in the figure and consists of two semicircles, $x^2+y^2=1$ and $x^2+z^2=1$, lying on the unit sphere. A portion of the sphere is shown as the shaded region.