- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED <u>AND</u> UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/041724 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) $\displaystyle\int_1^2 \int_0^{\sqrt{4-y^2}} 2x^2y^2 \, \mathrm{d}x \, \mathrm{d}y = \int_0^{\sqrt{3}} \int_1^{\sqrt{4-x^2}} 2x^2y^2 \, \mathrm{d}y \, \mathrm{d}x.$

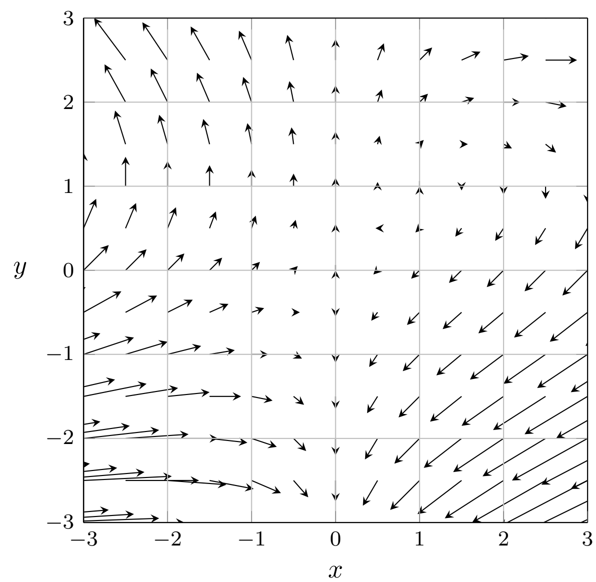
   (b) $\displaystyle\int_{-\sqrt{8-x^2}}^{x} \int_{-2}^{0} \sqrt{x^2+y^2} \, \mathrm{d}y \, \mathrm{d}x + \int_{-\sqrt{8-x^2}}^{-x} \int_0^2 \sqrt{x^2+y^2} \, \mathrm{d}y \, \mathrm{d}x = \int_0^{2\sqrt{2}} \int_{5\pi/4}^{7\pi/4} r \, \mathrm{d}\theta \, \mathrm{d}r$

   (c) The surface area of the first octant portion of the hyperboloid of two sheets described by $x^2 + y^2 - z^2 = -9,\ 3 \le z \le 5$, is

   $$\int_0^4 \int_0^{\sqrt{16-x^2}} \frac{\sqrt{2x^2+2y^2+9}}{\sqrt{x^2+y^2+9}} \, \mathrm{d}y \, \mathrm{d}x$$

   (d) The points $(x, y, z) = (-1, 1, -\sqrt{2})$ and $(\rho, \theta, \phi) = \left(2, -\frac{\pi}{4}, \frac{3\pi}{4}\right)$ describe the same point in space.

   (e) The vector field $\mathbf{V} = (x - xy)\,\mathbf{i} + (x - y)\,\mathbf{j}$ is shown in the accompanying figure.



2. [2350/041724 (16 pts)] A wire with charge density $q(x, y) = y^2$ is in the shape of the graph of $y = e^x, 0 \le x \le 1$. Find the total charge on wire.

CONTINUED ON REVERSE

3. [2350/041724 (34 pts)] A thin metal plate is in the shape, $\mathcal{R}$, of a triangle with vertices $(0,0),(1,2),(2,3)$. The metal's density is given by $\delta(x,y) = \sqrt{4x-2y}(-2x+2y)$. We need to find the coordinates of the center of mass of the plate. We will use the Change of Variables theorem to accomplish this endeavor using the linear transformation $u = 4x - 2y, v = -2x + 2y$.

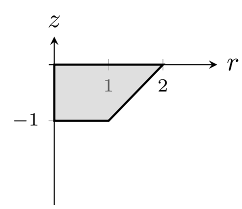
   (a) (6 pts) Find the transformations of the vertices of the triangle.

   (b) (6 pts) Recalling that linear transformations take lines into lines, draw the region of integration, $\mathcal{S}$, in the uv-plane, using the information from part (a). Be sure to label important points.

   (c) (18 pts) Find the mass of the plate.

   (d) (4 pts) Given that the moments with respect to the x- and y-axes are $\dfrac{104\sqrt{2}}{135}$ and $\dfrac{64\sqrt{2}}{135}$, respectively, find the coordinates of the center of mass of the plate. Note: $104 = (8)(13); 135 = (3)(45)$.

4. [2350/041724 (20 pts)] We need to evaluate $I = \iiint_{\mathcal{E}} z\left(x^2+y^2\right)^{3/2}\,\mathrm{d}V$ where the projection of $\mathcal{E}$ onto an rz-plane (constant θ) is shown in the following figure. The three dimensional solid region $\mathcal{E}$ is below the *second quadrant* of the xy-plane. In the parts that follow, set up, **do not evaluate**, integral(s) to compute I using the order of integration shown. Your limits must give the solid region as described, not using any potential symmetries. Simplify your integrands.



   (a) $\mathrm{d}\theta\,\mathrm{d}r\,\mathrm{d}z$

   (b) $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

   (c) $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

5. [2350/041724 (20 pts)] Let $\psi(x,y)$ be a function with continuous second partial derivatives and let $\mathbf{V} = \mathbf{k} \times \nabla\psi$ be a two-dimensional vector field.

   (a) (7 pts) Compute the divergence of $\mathbf{V}$.

   (b) (7 pts) Compute the curl of $\mathbf{V}$.

   (c) (3 pts) Is $\mathbf{V}$ incompressible? Justify your answer.

   (d) (3 pts) Is $\mathbf{V}$ irrotational? Justify your answer.