- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”× 11” crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: ”I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/031324 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The domain of $w(x,y,z) = \ln(x+1)\sqrt{z^2-y}$ is $\left\{(x,y,z) \in \mathbb{R}^3 \,\middle|\, x > 0, y \le z^2\right\}$.

   (b) The smallest (most negative) rate of change of $f(x,y,z) = xyz$ at the point $(1,2,3)$ is -7.

   (c) The function $g(x,y) = \begin{cases} \frac{x^2-4y^2}{x+2y} & (x,y) \ne (2,-1) \\ -4 & (x,y) = (2,-1) \end{cases}$ is continuous at $(2,-1)$.

   (d) The tangent plane to the surface $yz + y - xe^z - e^{x+1} = 1$ at the point $(-1,1,0)$ is $-2x + y + 2z = 3$.

   (e) The level curves of any plane that is neither vertical nor horizontal are lines.

2. [2350/031324 (18 pts)] Parts (a) and (b) are not related.

   (a) (10 pts) Find the simplified quadratic (second degree) Taylor polynomial, T_2, for $f(x,y) = \sin(xy)$ centered at $(\pi,-1)$.

   (b) (8 pts) A group of your friends has a second degree Taylor polynomial for some function, $f(x,y)$, centered at the point $\left(\frac{1}{2},1\right)$. They want to use the polynomial to approximate the function in the region $\left|x-\frac{1}{2}\right| \le \frac{1}{2}, |y-1| \le 1$. In addition, they have provided you with the following information:

   $$f_{xxx} = -3xy \qquad f_{xxy} = 3xy \qquad f_{xyy} = -7xy \qquad f_{yyy} = 4xy$$

   Tell your friends how much error they can expect in the approximation.

3. [2350/031324 (34 pts)] Kalkk3 Regional Park consists of the boundary and interior of the triangle described by $x=0, y=0$ and $x+y=9$. The elevation of the park above/below mean sea level $(h=0)$ is given by $h(x,y) = 2xy - \frac{2}{3}x^3 - y^2 + 100$. The editors of a travel brochure for the park want information about the various landforms in the park. Park surveyors have told you the following: maximum and minimum elevations along the southern $(y=0)$ border are 100 at $(0,0)$ and -386 at $(9,0)$, respectively, and along the western boundary $(x=0)$ they are 100 at $(0,0)$ and 19 at $(0,9)$.

   (a) (15 pts) Are there any saddles or local hills or valleys in the park? If so, what are their elevations and locations?

   (b) (15 pts) Use Lagrange Multipliers to find the maximum and minimum elevations and their locations along the remaining border, $x+y=9$.

   (c) (4 pts) If visitors to the park want to explore the highest and lowest points in the park, what points should they visit and what will their elevation be there?

4. [2350/031324 (22 pts)] The temperature in a region of space is given by $T(x,y,z) = 1000 + x^2 + y^2 + z^2$. The super-duper Rate-O-Change meter on board your spaceship gives a readout of the instantaneous rate of change of temperature with respect to any variable you enter into it. To receive credit, you must use Calculus 3 concepts to answer this question. Be careful with your notation.

   (a) (10 pts) Suppose you know the following information about the path of your spaceship: $\mathbf{r}(1) = 2\,\mathbf{i} + \mathbf{k}$, $\mathbf{r}'(1) = 2\,\mathbf{i} + 2\pi\,\mathbf{j}$. What does your meter read when you enter t into it and you are at the point $(2,0,1)$?

   (b) (12 pts) Now suppose your spaceship's position is given by $x(u,v,w) = v^2 + w^2$, $y(u,v,w) = \ln(uv)$, $z(u,v,w) = e^{2u+4v}$. What does your meter read if you enter u into it when $u=2$, $v=\frac{1}{2}$ and $w=1$?

5. [2350/031324 (16 pts)] The body mass index, B, as a function of weight, W (kg), and height, H (m), is given by $B = W/H^2$. Suppose that for a 2 m tall, 100 kg person you know that the height measurement is 0.01 m too high. If you want the body mass index to have an error no greater than 0.25, use differentials to determine the maximum error that can be present in the weight measurement.