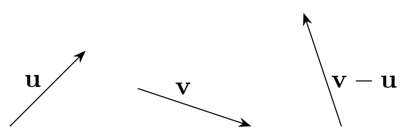
- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/021424 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) If two nonzero vectors of equal magnitude, $\mathbf{A}$ and $\mathbf{B}$, are such that $\mathbf{A}\cdot\mathbf{B} = \|\mathbf{A}\times\mathbf{B}\|$, the the angle between the vectors is $\pi/4$.

   (b) The following figure is geometrically correct.



   (c) The vectors $\mathbf{u} = \mathbf{i} + 5\,\mathbf{j} - 2\,\mathbf{k}$, $\mathbf{v} = 3\,\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = 5\,\mathbf{i} + 9\,\mathbf{j} - 4\,\mathbf{k}$ are coplanar.

   (d) The normal plane to the curve $\mathbf{r}(t) = t^3\,\mathbf{i} + 3t\,\mathbf{j} + t^4\,\mathbf{k}$ is parallel to the plane $3x + 3y - 4z = 2$ at the point $(-1, -3, 1)$.

   (e) $(\mathbf{a}\cdot\mathbf{b}) + \mathbf{c} = \mathbf{a}\cdot(\mathbf{b}+\mathbf{c})$

2. [2350/021424 (26 pts)] The following parts (a), (b) and (c) are not related.

   (a) (6 pts) Let $\mathbf{A}$ be the vector from the point $(2, -2, 4)$ to the point $(-2, 3, 5)$. Find a vector $\mathbf{D}$ having magnitude $\frac{1}{3}$ in the same direction as $\mathbf{A}$.

   (b) (12 pts) Let $\mathbf{a} = \mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = \mathbf{j} - 2\,\mathbf{k}$.

      i. (6 pts) Find the area of the parallelogram formed by $\mathbf{a}$ and $\mathbf{c}$.

      ii. (6 pts) Find the vector projection of $\mathbf{a}$ onto $\mathbf{c}$.

   (c) (8 pts) Consider the equation $2x^2 + \alpha y^2 + z^2 - 2y - 4z + \beta = 0$.

      i. (2 pts) If $\alpha = 1$, find all values of β, if any, such that the equation describes an ellipsoid.

      ii. (6 pts) If $\alpha = -1$, find all values of β, if any, such that the equation describes a

      A. hyperboloid of one sheet   B. hyperboloid of two sheets   C. cone

3. [2350/021424 (28 pts)] The path of a model rocket above ground (where $z = 0$) is given by

$$\mathbf{r}(t) = t^2\,\mathbf{i} + (\cos t + t\sin t)\,\mathbf{j} + (\sin t - t\cos t + 4\pi)\,\mathbf{k},\ t \geq 0$$

   The label on the rocket engine claims that it has enough fuel to allow the rocket to travel $2\pi^2\sqrt{5}$ units along its path.

   (a) (2 pts) Show that the velocity of the rocket is $2t\,\mathbf{i} + t\cos t\,\mathbf{j} + t\sin t\,\mathbf{k}$.

   (b) (8 pts) When will the fuel run out, assuming that the fuel begins burning when $t = 0$?

   (c) (4 pts) Find the coordinates of the rocket when the fuel has been completely consumed.

   (d) (4 pts) Assuming the rocket is on the launchpad at $t = 0$, how far from the launchpad is the rocket when the fuel runs out?

   (e) (10 pts) Once the fuel is consumed, the rocket initially moves in the direction it was moving when the fuel ran out but is acted on by a gravitational acceleration of $\mathbf{a} = \langle 0, 0, -10\rangle$. How long after the fuel is gone does it take the rocket to reach the ground? Hint: It may simplify things to consider a new time parameter, τ, equal to zero when the fuel is depleted.

CONTINUED ON THE BACK

4. [2350/021424 (20 pts)] Consider the two points $P(1,0,2)$ and $Q(-1,2,0)$ and the plane M described by $2x-4y+z=10$.

   (a) (10 pts) Find the equation of the plane containing the points P and Q and perpendicular to the plane M. Write your answer in the standard form $ax+by+cz=d$.

   (b) (10 pts) Find the symmetric equations of the line orthogonal to M and passing through point Q.

5. [2350/021424 (16 pts)] A particle is moving along the path $\mathbf{r}(t) = e^t\,\mathbf{i} + 2\,\mathbf{j} + e^{-t}\,\mathbf{k}$.

   (a) (4 pts) Without doing any computations, briefly explain why the unit binormal, $\mathbf{B}$, is always parallel to the y-axis.

   (b) (12 pts) Recalling that the particle's velocity is a vector and its change, the acceleration, can be decomposed into two orthogonal components, answer the following questions.

      i. (5 pts) Find the rate of change of the direction of the particle as a function of t.

      ii. (7 pts) What is the speed of the particle at the point where its speed is not changing?