- This exam is worth 150 points and has 6 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on two sides.

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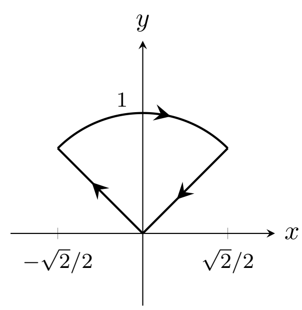
0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/050823 (46 pts)] A wire is in the shape of the curve $\mathcal{C}$ given by $\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + t^2\,\mathbf{k}$, $-1 \le t \le 2$.
   (a) [9 pts] Does the wire intersect the plane that contains the points $(1, 2, 0)$, $(0, 0, 3)$, $(0, -1, 4)$? If so, find the point of intersection. If not, explain why not.
   (b) [5 pts] What is the curvature of the wire when $t = 0$?
   (c) [15 pts] If the charge density (Coulomb per unit length) on the wire is given by $q(x, y, z) = \dfrac{5xyz}{\sqrt{2 + 4xy}}$, find the total charge on the wire.
   (d) [17 pts] Answer the following questions if the wire is immersed in an electric field, $\mathbf{E}$, given by
   $$\mathbf{E} = -x\cos y\,\mathbf{i} + y\cos x\,\mathbf{j} + xye^{x-y+z^2}\,\mathbf{k}$$
      i. [2 pts] Is the electric field incompressible? Justify your answer.
      ii. [15 pts] Find the work done moving a charged particle along the wire. Hint: $\mathbf{E}$ is not irrotational.

2. [2350/050823 (20 pts)] Consider the oriented curve, $\mathcal{C}$, shown in the figure (the curved portion is an arc of the unit circle). Compute the circulation of $\mathbf{V}$ on $\mathcal{C}$ where $\mathbf{V} = \left(-16y + \sin x^2\right)\mathbf{i} + \left(4e^y + 3x^2\right)\mathbf{j}$.



3. [2350/050823 (22 pts)] Consider the three dimensional solid, $\mathcal{E}$, below the surface $\mathcal{S}_1 : \rho = 2\cos\phi$ and above the surface $\mathcal{S}_2 : \phi = \dfrac{\pi}{4}$. Let $\mathbf{F} = y\,\mathbf{i} + x\,\mathbf{j} + \left(6z\tan^{-1}\dfrac{y}{x}\right)\mathbf{k}$. Recall that $\tan\theta = \dfrac{y}{x}$.
   (a) [2 pts] Name the surfaces, $\mathcal{S}_1$ and $\mathcal{S}_2$.
   (b) [20 pts] Find the outward flux of $\mathbf{F}$ through the boundary of $\mathcal{E}$ $(\partial\mathcal{E} = \mathcal{S}_1 \cup \mathcal{S}_2)$.

4. [2350/050823 (20 pts)] Compute $\displaystyle\int_{\mathcal{C}} P\,\mathrm{d}x + Q\,\mathrm{d}y + R\,\mathrm{d}z$ where $\mathbf{F} = \langle P, Q, R\rangle = y\,\mathbf{i} + x^2\,\mathbf{j} + z\,\mathbf{k}$ by evaluating an appropriate surface integral. $\mathcal{C}$ is the boundary of the portion of the plane $x + y + 5z = 1$ in the first octant, oriented counterclockwise when viewed from above.

5. [2350/050823 (15 pts)] The temperature in a certain region of the xy plane is given by $T(x, y) = 2x^2 + 2y^2 + x - y$. You are walking along the circle $x^2 + y^2 = 8$. Use Lagrange Multipliers to find the maximum and minimum temperatures you experience during your walk.

CONTINUED ON REVERSE

6. [2350/050823 (27 pts)] On a separate page in your bluebook, write the letters (a) through (i) in a column. Then for the following questions, write the word **TRUE** or **FALSE** next to each letter, as appropriate. No partial credit given and no work need be shown. If you do any work to come up with your answers, please do it elsewhere - do not include it in your list of answers (this helps with grading).

   (a) The flow along any curve $\mathcal{C}$ between the origin and $(r, \theta, z) = \left(\sqrt{2}, \frac{\pi}{4}, 1\right)$ of the vector field $\mathbf{F} = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ is 1.

   (b) The function $z = (x-1)(y+1)$ has a local maximum at $(1, -1)$.

   (c) The second order Taylor polynomial centered at $(0,0)$ of the function $f(x,y) = xy$ is $f(x,y)$.

   (d) The Extreme Value Theorem guarantees that $f(x,y) = \dfrac{xy}{1+x^2+y^2}$ attains an absolute maximum on the region $x^2 + y^2 < 2$.

   (e) The binormal vector, $\mathbf{B}$, of any curve lying entirely in the xz-plane is parallel to the y-axis.

   (f) A particle traveling along the path $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + e^t\,\mathbf{k},\ t \geq 0$ never slows down.

   (g) The function $f(x,y) = x^2 - xy + y^2$ increases the fastest at the point $(2,1)$ in the direction of $-3\,\mathbf{i}$.

   (h) The level surfaces of $w(x,y,z) = x^2 - y^2 + 2z^2$ are hyperbolic paraboloids.

   (i) The function $f(x,y) = \begin{cases} 0 & (x,y) = (0,0) \\ \dfrac{x^3y}{x^4+y^4} & (x,y) \neq (0,0) \end{cases}$ is continuous throughout $\mathbb{R}^2$.