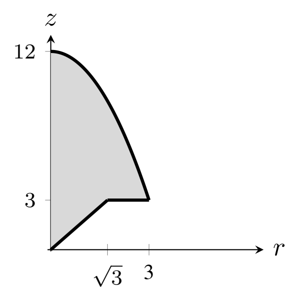
- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5”$\times$ 11” crib sheet with writing on one side.

NOTE: Any integrals that need to be evaluated will require integration techniques no more complicated than u-substitution.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: ”I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/041923 (22 pts)] You are in charge of making three-dimensional stoppers for glass bottles. The density of the material from which the stoppers are made is $\delta(x, y, z) = x^2 + y^2 + 7$. A cross section of half of the stopper is depicted in the following rz–plane (constant θ plane). The curved portion of the stopper is $x^2 + y^2 + z = 12$.



   (a) [8 pts] Set up, **but do not evaluate**, the appropriate integral(s) that will give the mass of the stopper using cylindrical coordinates and the order $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$.

   (b) [14 pts] Now suppose that only the curved portion of the stopper is replaced with a portion of the sphere of radius 3, centered at $(x, y, z) = (0, 0, 3)$, while the rest of the stopper's shape remains the same.

      i. [6 pts] Make a sketch of this new stopper in the rz–plane (constant θ plane). Be sure to label important points in your sketch.

      ii. [8 pts] Using spherical coordinates and the order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$, set up, **but do not evaluate**, the appropriate integral(s) to find the mass of this new stopper.

2. [2350/041923 (20 pts)] The charge density on a metal plate is given by $q(x, y) = \sqrt{xy}$ coulombs per square meter. The plate occupies the first quadrant region bounded by $xy = 1$, $xy = 9$, $y/x = 1$, and $y/x = 4$. Use the change of variables $u = xy$ and $v = y/x$ to find the total charge on the plate. Be sure to include appropriate units.

3. [2350/041923 (28 pts)] The depth of a lake is described by the function

$$s(x, y) = \frac{-1}{1 + (x/2)^2 + (y/2)^2}$$

   where all measurements are in kilometers. The lake's center is at the origin of a Cartesian coordinate system whose positive y-axis points north and positive x-axis points east with the lake's surface at $z = 0$. In order to conduct some underwater experiments, your lab supervisor needs to know the average depth of that part of the lake lying to the west of the lake's center, bounded by the lines $y = \sqrt{3}x$ and $y = -x/\sqrt{3}$ and the circles of radius 2 and 4 kilometers from the center of the lake.

   (a) [8 pts] Make a sketch in the plane $z = 0$ of that part of the lake whose average depth you are required to find and describe it using appropriate inequalities. Be sure to label important points in your sketch.

   (b) [20 pts] What number should you report to the lab supervisor? Be sure to include the appropriate units.

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4. [2350/041923 (20 pts)] Let $\mathcal{S}$ be a thin lamina consisting of the portion of $2z = \sqrt{1+x^2+y^2}$ lying inside the cylinder of radius 2 aligned along the z–axis. Find the mass (including units) of $\mathcal{S}$ if the density (g/cm^2) of the material that comprises $\mathcal{S}$ is

$$\delta(x,y,z) = \frac{z(x+5)}{\sqrt{5x^2+5y^2+4}}$$

5. [2350/041923 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

(a) Considering points only in the first octant, if $\mathbf{F} = \frac{1}{x}\,\mathbf{i} + \frac{1}{y}\,\mathbf{j} + \frac{1}{z}\,\mathbf{k}$ and $f = \ln(xyz)$, then $\mathbf{F}$ is conservative.

(b) $\mathbf{G} = x\,\mathbf{i} + xy\,\mathbf{j} - \mathbf{k}$ is irrotational.

(c) If $\mathbf{H} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and $\phi(x,y,z)$ is a function with continuous partial derivatives of all orders, then $\nabla\cdot(\phi\mathbf{H}) = 3\phi + \mathbf{H}\cdot\nabla\phi$.

(d) If $f(x,y,z) = xyz^{-1}$, then the divergence of the gradient of f is 0 for all $x, y, z > 0$.

(e) The vector field $\mathbf{V} = \frac{y}{\sqrt{x^2+y^2}}\,\mathbf{i} + \frac{x}{\sqrt{x^2+y^2}}\,\mathbf{j}$ is shown in the accompanying figure.