- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one side.

---

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/031523 (26 pts)] The following parts (a), (b), and (c) are not related.

   (a) [14 pts] Let $F(x, y, z) = \left(1 - x^2\right)^{1/2} e^{y^2+z^2}$.

      i. [4 pts] Find the domain of the function.

      ii. [4 pts] Does the level surface $F(x, y, z) = -1$ exist? (answer YES or NO). If yes, find it. If no, explain why not.

      iii. [6 pts] Find the direction vector of the normal line to the surface $F(x, y, z) = \frac{\sqrt{2}e}{2}$ at the point $\left(\frac{1}{\sqrt{2}}, 0, 1\right)$.

   (b) [5 pts] Evaluate $\displaystyle\lim_{(x,y)\to(2,-2)} \frac{x^2 - y^2}{x + y}$ or explain why the limit does not exist.

   (c) [7 pts] Find the rate of change of w with respect to v at the point $(u, v) = (-1, 2)$ if $w = xy + \ln z$, $x = u^2/v$, $y = u + v$ and $z = \cos u$.

2. [2350/031523 (21 pts)] The following parts (a) and (b) are not related.

   (a) [8 pts] The magnetic field in the first octant of $\mathbb{R}^3$ is given by $B = \ln(xyz)$. To recharge your spaceship's fuel cells in the most efficient way possible, you want to guide your ship in the direction that produces the greatest rate of change of the magnetic field with respect to distance. When you are at the point $(1, 1, 2)$, find the direction you should aim your ship and determine the corresponding rate of change of the magnetic field.

   (b) [13 pts] A rectangular metal plate occupies the region $|x| \leq 4, |y| \leq 6$. Its thickness is given by the continuous function $h(x, y)$. Use the following information about $h(x, y)$ to answer the given questions below.

$$\frac{\partial h}{\partial x} = -2xe^y \qquad \frac{\partial h}{\partial y} = e^y(2y + y^2 - x^2)$$

$$\frac{\partial^2 h}{\partial x^2} = -2e^y \qquad \frac{\partial^2 h}{\partial y^2} = e^y\left(2 + 4y + y^2 - x^2\right) \qquad \frac{\partial^2 h}{\partial x \partial y} = -2xe^y$$

      i. [8 pts] Are there any points in the plate that are locally thicker or thinner than their nearby surroundings? If so, where are they? If not, explain why not.

      ii. [5 pts] Is there a thinnest part of the plate? Do not find it, simply answer YES or NO and give a brief explanation justifying your answer.

3. [2350/031523 (25 pts)] Let $g(x, y) = \cos(\pi xy) + xy^2$.

   (a) [8 pts] In what direction(s) will you have to move to follow a level curve at the point $(1, 1)$? Express your answer(s) as unit vector(s).

   (b) [7 pts] At the point $(1, 1)$, is g more sensitive to small changes in x or to small changes in y? Explain using differentials.

   (c) [10 pts] Suppose $g(x, y)$ represents the temperature at the point (x, y). If you are walking along the path $\mathbf{r}(t) = \dfrac{1}{\sqrt{t}}\,\mathbf{i} + \dfrac{t^2}{16}\,\mathbf{j}$, is the temperature increasing or decreasing with respect to time as you pass through the point $\left(\frac{1}{2}, 1\right)$? At what rate?

4. [2350/031523 (16 pts)] A skateboard park has a new track in the form of the hyperbola $\frac{1}{4}x^2 - y^2 = 12$. You are standing at the point $(x, y) = (0, 10)$ watching your friends skate on the track. Using Lagrange Multipliers, determine the closest your friends will get to you. Where will they be when this occurs?

CONTINUED ON REVERSE

5. [2350/031523 (12 pts)] The following parts (a) and (b) are not related.

(a) [7 pts] Find the second order/quadratic Taylor approximation of $f(x,y) = e^{-(x^2+2y)}$ centered at the point $\left(1, -\frac{1}{2}\right)$. Do not simplify your answer.

(b) [5 pts] Suppose you have the following information about a function, $g(x,y)$:

$$g_{xx}(x,y) = \frac{1}{x^2+y^2+2}, \qquad g_{xy} = \frac{-3}{x^2+y^2+2}, \qquad g_{yy} = \frac{20x}{x^2+y^2+2}$$

Using this information, find an upper bound on the error in the first order/linear Taylor approximation for $g(x,y)$ centered at the origin if $|x| \leq 0.1$ and $|y| \leq 0.3$. Do not find the Taylor approximation (you can't actually).