- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"$\times$ 11" crib sheet with writing on one side.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/021523 (15 pts)] Han Solo and Chewbacca are piloting the Millennium Falcon along the path given by

$$\mathbf{r}(t) = \cos(\pi t)\,\mathbf{i} + \sin(\pi t)\,\mathbf{j} + (4t - t^2)\,\mathbf{k}, \quad t \geq 0$$

When $t = 3$, Chewy fires a laser beam from the front of the ship that travels forward in a straight line. Find the coordinates of the point where the laser beam penetrates the xy-plane.

2. [2350/021523 (12 pts)] Consider the equation $-\frac{1}{4}z^2 - 8y^2 + x^2 + 2z - 20 = 0$

   (a) [4 pts] Name the surface, providing justification for your answer.

   (b) [4 pts] Does the surface intersect the yz-plane? Justify your answer.

   (c) [4 pts] Name the conic section of the trace when $x = -\sqrt{32}$, providing justification for your answer.

3. [2350/021523 (15 pts)] Find the equation of the plane that is perpendicular to the plane $2z = 5x + 4y$ and contains the line with symmetric equations $-x = \dfrac{y+2}{5} = \dfrac{z-5}{-4}$. Write your final answer in the form $ax + by + cz = d$.

4. [2350/021523 (46 pts)] A scorpion is crawling on a shelf located 2 meters above the floor in a room. Its path is given by

$$\mathbf{r}(t) = \frac{t^3}{3}\mathbf{i} + \frac{t^2}{2}\mathbf{j} + 2\,\mathbf{k} \quad t \geq 0,$$

with distances measured in meters. Answer **ALL** of the following questions for $t = 1$ second.

   (a) [3 pts] Where is the scorpion?

   (b) [4 pts] How fast is the scorpion crawling? (Include units in your answer)

   (c) [8 pts] Briefly describe in words (*i.e.* DO NOT COMPUTE) what the following two quantities represent physically in terms of the scorpion's path:

   i. $\sqrt{\mathbf{r}(t) \cdot \mathbf{r}(t)}$ ii. $\displaystyle\int_0^t \sqrt{\mathbf{r}'(u) \cdot \mathbf{r}'(u)}\,\mathrm{d}u$

   (d) [5 pts] Find the scorpion's unit tangent vector, $\mathbf{T}$.

   (e) [5 pts] Find the unit normal to the scorpion's path, which for this curve can be accomplished by computing $\mathbf{T} \times \mathbf{k}$.

   (f) [5 pts] Find the binormal vector to the scorpion's path by calculating $\mathbf{T} \times \mathbf{N}$.

   (g) [4 pts] Find the equation of the scorpion's osculating plane.

   (h) [4 pts] In your bluebook, draw an xyz-coordinate system according to the right hand rule such that the positive z-axis is perpendicular to and out of your paper and your paper is the plane $z = 2$. Then draw $\mathbf{T}(1)$ and $\mathbf{N}(1)$ at the appropriately labeled point. Be sure to label the vectors correctly.

   (i) [4 pts] How fast is the scorpion's speed changing? (Include units in your answer)

   (j) [4 pts] Does the scorpion's acceleration possess a component normal to its path? If so, find its magnitude, including units in your answer. If not, explain why not.

5. [2350/021523 (12 pts)] Consider the force given by $\mathbf{F} = 2\,\mathbf{i} + 7\,\mathbf{j} + 3\,\mathbf{k}$ Newtons.

   (a) [6 pts] Suppose you are moving in a straight line in the direction of $\mathbf{v} = 6\,\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$. Calculate the work done by the force if you continue moving along the line a total distance of 6 meters.

   (b) [6 pts] Consider a beam mounted at the point P that can rotate around that point. The force, $\mathbf{F}$, is applied to the beam at an angle of 30 degrees, resulting in a torque of magnitude 30 Newton-m. How far from P was the force applied?