# APPM 2350—Final Exam

*Saturday April 30th, 10:30am-1pm 2022*

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This exam has 5 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5×11-in page of notes (TWO sided). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

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**Problem 1** (30 points) The following questions are not related:

(a) Find the value(s) of $a$ and $b$ where the function

$$I(a,b) = \int_a^b (-2x^2 + 10x - 12)\, dx$$

has a local maximum. Be sure to support your answer using Calculus 3 concepts.

(b) Consider the function

$$f(x,y,z) = x^2y - z\cos(y)$$

Use a directional derivative to approximate how much $f$ changes if one moves a distance $0.1$ from the point $(4,0,3)$ straight toward the origin.

(c) Arrange the following three double integrals in order from least to greatest **and explain/justify your reasoning**:

$$\int_0^2 \int_0^1 e^{x^2+y^2}\, dx\, dy, \qquad \int_0^2 \int_0^{2-\frac{y}{2}} e^{x^2+y^2}\, dx\, dy, \qquad \int_0^2 \int_0^{1-\frac{y}{2}} e^{x^2+y^2}\, dx\, dy$$

**Problem 2** (30 points)
Given the force vector field

$$\mathbf{F}(x,y,z) = 2y\mathbf{i} + 3z\mathbf{j} - x\mathbf{k}$$

Consider the plane $\mathcal{P}$ that passes through the points $(1,1,3)$, $(3,0,1)$ and $(-2,2,7)$. Let $\mathcal{C}$ be any closed circular path with radius $a$ that lies in the plane $\mathcal{P}$, oriented counterclockwise when viewed from above (that is, when viewed from the positive z-axis looking down). Note, the circular path $\mathcal{C}$ does not necessarily pass through the given points. (It is tricky to parameterize the path $\mathcal{C}$, so **don't try to parameterize** it during this exam).

(a) Without parameterizing the path, what is the curvature of the path $\mathcal{C}$?
(b) Without parameterizing the path, what is the unit binormal, $\hat{\mathbf{B}}$, to the path $\mathcal{C}$?
(c) Without parameterizing the path, find the work done by $\mathbf{F}$ once around $\mathcal{C}$.

**Problem 3** (30 pts)
A velocity field is given by

$$\mathbf{F} = a(z + xe^y)\mathbf{i} + (bx^2 + cz)e^y\mathbf{j} + (\sin z + e^y)\mathbf{k}$$

(a) For what values of $a, b$ and $c$ will $\mathbf{F}$ be conservative? Be sure to justify your answer and double check your work.
(b) Using the values of $a, b$ and $c$ you found above, find the flow of $\mathbf{F}$ along the straight line path starting at $(0,0,0)$ and ending at $(1,0,\pi)$ using an appropriate Calculus 3 theorem.
(c) Verify your answer in part (b) by direct computation (i.e. by evaluating a line integral).

CONT'D ON REVERSE

**Problem 4** (20 pts)
The following questions are not related:

(a) Suppose

$$\int_{\mathcal{C}} 4y\,dx + 7x\,dy = 13$$

where $\mathcal{C}$ is a simple, smooth curve oriented counter-clockwise in the $xy$-plane that encloses the region $\mathcal{R}$. Given only this information, is it possible to find the area of $\mathcal{R}$? If so, find it and justify your reasoning. If not, explain what additional information you'd need.

(b) Let $\iint_{\mathcal{R}} dA$ give the area of a region $\mathcal{R}$ in the first quadrant of the $xy$-plane. (Note, this region is not related to the region $\mathcal{R}$ in part (a)). You are interested in finding the **volume** $V$**, generated by revolving** $\mathcal{R}$ **about the x- axis**. If $\iint_{\mathcal{R}} g(x,y)\,dA$ is the integral that calculates the **volume** $V$, determine the integrand $g(x,y)$.

**Problem 5** (40 pts)
Consider the 3D solid object $\mathcal{E}$ that is bounded on the top by $z = 2$, on the bottom by $z = 0$ and on the sides by $x^2 + y^2 + z^2 = 8$.
Let

$$\mathbf{G} = y\mathbf{i} - x\mathbf{j} + 3z\mathbf{k}$$

(a) Sketch and shade a cross section of the object in the $rz$-plane. Label axes and any intercepts.
(b) Calculate the volume of the object.
(c) Calculate the outward flux of the vector field $\mathbf{G}$ through the entire surface of the object $\mathcal{E}$ using an appropriate Calculus 3 theorem.
(d) Verify your answer to part (c) by separately calculating the flux through each part of the bounding surface (i.e. the top, the bottom and the side) and adding them together.

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End Of Exam: Have a great summer!

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