APPM 2350—Exam 3

Wednesday April 13th, 6:30pm-8pm 2022

This exam has 4 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (20 points)
Suppose the temperature at any point in the xy-plane is given by

$$T(x, y) = xe^{-y^2}$$

Suppose the area of a region $\mathcal{R}$ is given by

$$\int_0^3 \int_{x^2}^9 dy\, dx$$

(a) Sketch **and shade** the region $\mathcal{R}$ and clearly label your axes and any intercepts.
(b) Find the **average temperature** on the region $\mathcal{R}$. Fully simplify your final answer.

Problem 2 (30 points)
The volume of an object is given by

$$\int_0^{\pi/4} \int_0^1 \int_{\sqrt{3}}^3 r\, dz\, dr\, d\theta + \int_0^{\pi/4} \int_1^{\sqrt{3}} \int_{r\sqrt{3}}^3 r\, dz\, dr\, d\theta$$

(a) Sketch and shade a 2D cross section of the object in the rz-plane (for any θ such that $0 \le \theta \le \pi/4$). Label the (r, z) coordinates of all corners on the cross section.
(b) Set up, **do not evaluate** equivalent integral(s) to find the volume of the object using:
  (i) Cylindrical coordinates in the order $dr\, dz\, d\theta$
  (ii) Spherical coordinates in the order $d\rho\, d\phi\, d\theta$

Problem 3 (30 points)
The following parts are not related:

(a) A thin wire lies along the curve $x = \frac{2}{3}(y-1)^{3/2}, \quad 1 \le y \le 4,$ in the xy-plane. Find the **y-component of the centroid** of the wire. Fully simplify your answer.
(b) Let S be any region (e.g. a rectangle, circle, or any other shape) on the plane $3x - 4y + 2z = 6$ that has a surface area equal to 5. Let $\mathcal{R}$ be the projection of the region S onto the **yz-plane**. Find the **area of** $\mathcal{R}$.

Problem 4 (20 points)
Consider the region, $\mathcal{R}$, that is bounded by

$$y = \sqrt{x}, \quad y = \sqrt{x} + 2, \quad y = 4 - \sqrt{x}, \quad y = 6 - \sqrt{x}$$

where x and y are measured in meters.
A sprinkler sprays water on the region $\mathcal{R}$ in such a way that the depth of water (in meters) that reaches the point (x, y) in 1 hour is given by

$$g(x, y) = \frac{e^{(y-\sqrt{x})}}{20\sqrt{x}}$$

Use an appropriate uv-transformation to **find the total volume of water** the sprinkler sprays on the region $\mathcal{R}$ in 1 hr. Fully simplify your final answer.

End Of Exam