APPM 2350—Exam 2

Wednesday March 9th, 6:30pm-8pm 2022

This exam has 4 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (30 points)
The temperature at any point in the xy-plane is given by

$$T(x,y) = x^2 + 3y^2 - 2y$$

(a) Find and classify any local max, min or saddle points of $T(x,y)$.
(b) Use Lagrange multipliers to find the location(s) of the hottest and coldest points on the ellipse $x^2 + 2y^2 = 8$.
(c) Given

$$x = 2 + \ln(\theta + 4z), \qquad y = r\cos(\pi r\theta) \qquad z = e^{sr}$$

Use the chain rule to find $\frac{\partial T}{\partial r}$. You can leave the variables x, y and z in your final answer without needing to substitute.

Problem 2 (20 points)
The following questions are not related:

(a) Let

$$h(x,y) = \frac{2x^2 - 2y^2}{x+y}$$

(i) The $\lim_{(x,y)\to(1,-1)} h(x,y)$ exists. Determine its value.
(ii) Is $h(x,y)$ continuous at $(1,-1)$? Justify your answer.

(b) Given

$$f(x,y,z) = \frac{1}{2}e^{4x^2 - y^2 + z^2}$$

(i) Give the equation of the level surface of $f(x,y,z)$ through the point $(1,6,4)$. Then classify (i.e. *give the official name of*) this surface.
(ii) Sketch the level surface you found in part (b)(i). Label your axes and label any intercepts on your sketches.

Problem 3 (28 points)
Suppose $g(x,y,z)$ is a continuous function with continuous partial derivatives for which

$$g(2,1,-3) = 6 \text{ and } \nabla g(2,1,-3) = \langle 5,-2,10\rangle$$

(a) Find the rate of change of g at the point $(x,y,z) = (2,1,-3)$ in the direction toward the point $(x,y,z) = (5,-1,-9)$.
(b) Use linearization to approximate $g(2.1, 0.9, -2.8)$.
(c) Let S be the surface $g(x,y,z) = 6$. Find the equation of the plane tangent to S at the point $(x,y,z) = (2,1,-3)$.
(d) Suppose the surface S in part (c) is the graph of a function, $z = f(x,y)$. Use linearization to approximate $f(2.1, 0.9)$.

CONT'D ON REVERSE

Problem 4 (22 points)
Let $f(x, y)$ be a continuous function with continuous partial derivatives.
Suppose the quadratic Taylor approximation of $f(x, y)$ centered at the point $(x, y) = (1, 0)$ is given by

$$Q(x, y) = \frac{1}{4}\pi - \frac{3}{4} + x + 2y - \frac{x^2}{4} - xy - y^2$$

For each part below, determine if you have enough information to determine the *exact quantities* described (not estimates) using only $Q(x, y)$ without any other information about $f(x, y)$. If so, find the quantity described. If not, explain what other information you would need.

(a) $f(2, 2)$
(b) $f_{xy}(1, 0)$
(c) The **value** of the smallest (i.e. most negative) derivative of $f(x, y)$ at the point $(1, 0)$ (out of all possible directions)
(d) $f_{xxx}(1, 0)$

End Of Exam
