APPM 2350—Exam 1

Wednesday Feb 9th, 6:30pm-8pm 2022

This exam has 5 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (26 points)

(a) An engineer measures three points $A = (1, 0, 3)$, $B = (3, 0, 4)$, and $C = (2, 1, 4)$ on a plane P. Find the equation of the plane P. Give your answer in standard (linear) form. (Before moving on, check your final answer by plugging in all 3 points and verifying they are on the plane you found).

(b) You decide to drill along a straight line starting at $Q = (4, 3, 0)$ and perpendicular to the plane P of part (a). Find the point (x, y, z) where the drill intersects the plane P.

(c) Let L be the line you drilled along in part (b), and let L' be the line through the points A and B of part (a). Are L and L' parallel, intersecting, or skew? Justify your answer.

Problem 2 (16 points) The following questions are not related:

(a) Suppose your roommate uses the force vector $-3\mathbf{i} + 5\mathbf{k}$ to push an object along a straight line path from $(x, y, z) = (3, -1, -7)$ to $(x, y, z) = (1, 5, -4)$. You use the force vector $\mathbf{F}$ to push an object along the same path, but in the optimal direction (i.e. parallel to the path) and you do the same work as your roommate. Find $\mathbf{F}$.

(b) Suppose the curve $\mathcal{C}$ is parameterized with respect to arclength by $\mathbf{r}(s)$. Given only this information, can you determine the distance along the curve $\mathcal{C}$ between $\mathbf{r}(3)$ and $\mathbf{r}(10)$? If so, give the distance (as a real number) and justify your answer. If you do not have enough information, explain what additional information you would need to complete this calculation.

Problem 3 (26 points)

A new drone designed for planet exploration is designed with a dual propulsion system with both fixed wing propellers and a rear thruster. Navigational systems on the drone track the direction of travel as

$$\hat{\mathbf{T}}(t) = \left\langle \frac{1}{1+2t^2}, \frac{2t}{1+2t^2}, 1 - \frac{1}{1+2t^2} \right\rangle$$

where t is given in seconds and the speed of the drone at time t is give by $f(t) = 1 + 2t^2 \quad \frac{miles}{sec}$

(a) Find the distance the drone travels along its path from $t = 1$ to $t = 2$.

(b) If the drone's position when $t = 1$ is $\langle 2, 1, -\frac{1}{3} \rangle$ find the position of the drone when $t = 3$.

(c) Find the (x, y, z) coordinate(s) of the drone's location(s) when $a_T = 8$.

Problem 4 (16 points)

You are managing a spacecraft that has been launched into space to fly to the space station. You have determined that the space station landing spot has the coordinates $(x, y, z) = (8, -2, 0)$. The spacecraft is currently at the point $(x, y, z) = (4, -6, 3)$ and traveling along a straight line in the direction of $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(a) How far is the spacecraft currently from the landing spot?

(b) Will the spacecraft hit the landing spot if it continues along its current trajectory? Explain why or why not. If not, find the (x, y, z) coordinates of the point on the spacecraft's path that is closest to the landing spot.

CONT'D ON REVERSE

Problem 5 (16 points)

The following questions are not related:

(a) Give an example of ONE vector-valued function $\mathbf{r}(t)$ that traces out the curve of intersection of the surfaces $x = 4y^2$ and $x^2 = 2z - 6y^2$

(b) Give an example of ONE vector-valued function, $\mathbf{r}(t)$, $t \geq 0$ with ALL of these specified properties (or explain why such a function does not exist):

- $\mathbf{r}(0) = \langle 0, 4, 0 \rangle$
- and $\kappa(t) = \frac{1}{4}$ for all $t \geq 0$
- and $\mathbf{B}(t) = \mathbf{i}$ for all $t \geq 0$
- and $a_T(t) \neq 0$ for $t > 0$

End Of Exam
