Show all your work and simplify your answers. Answers with no justification will receive no points.

**Problem 1 (24 points)**

Consider the integral \( \int \int_{R} (x^2 + y^2) \, dA \) where \( R \) is the region in the first quadrant given by

\[
R = \{(x, y) \mid 1 \leq x \leq \sqrt{2} \text{ and } x^2 + y^2 \leq 4 \text{ and } y \geq 0 \}.
\]

(a) Sketch and shade the region \( R \) in the first quadrant of the \( xy \)-plane. Label any intercepts.

(b) Set up the double integral(s) in terms of \( x \) and \( y \) with order of integration \( dy \, dx \). Do not evaluate.

(c) Set up the double integral(s) in terms of \( x \) and \( y \) with order of integration \( dx \, dy \). Do not evaluate.

(d) Set up the double integral(s) in terms of polar coordinates with order of integration \( dr \, d\theta \). Do not evaluate.

(e) Which version of the double integral would you rather compute and, more importantly, why? (Just explain which one you would prefer to actually evaluate, but do not evaluate the integral).

**Problem 2 (24 pts)**

Use an appropriate \( uv \)-transformation to evaluate the integral

\[
\int \int_{R} xe^{y-x^2} \, dA
\]

where \( R \) is the region in the first quadrant bounded by \( y = x^2, \ y = x^2 + 2, \ y = 2 - x^2, \) and \( y = 3 - x^2 \). Be sure to include a sketch of \( R \) and a sketch of the corresponding region \( S \) in the \( uv \)-plane in your solution.

**Problem 3 (36 points)**

Consider the following 2 spheres:

Sphere A: \( x^2 + y^2 + z^2 = 2 \)

Sphere B: \( x^2 + y^2 + (z - 1)^2 = 1 \)

Let \( E \) be the intersection of the regions inside both spheres (i.e., the 3D solid that’s common to both spheres). Suppose the temperature at any point in space is given by \( T(x, y, z) = x^2y^2z^2 \) degrees Fahrenheit.

(a) Sketch and shade the cross section of \( E \) in the \( rz \)-plane. Label any intercepts.

(b) Sketch and shade the projection of \( E \) onto the \( xy \)-plane. Label any intercepts.

(c) Set up integral(s) to find the volume of \( E \) using spherical coordinates in the order \( d\rho \, d\phi \, d\theta \). Do not evaluate the integral(s).

(d) Set up integral(s) to find the \( z \) component of the centroid of \( E \) using cylindrical coordinates in the order \( dz \, dr \, d\theta \). Do not evaluate the integral(s).

(e) Set up integral(s) to find the average temperature on the surface of the top hemisphere of Sphere A (use the coordinate system that leads to the fewest number and simplest integrals). Do not evaluate the integral(s).
Problem 4 (16 points)

The following questions are not related:

(a) The following double integral gives the volume of a 3D region $\mathcal{E}$. Sketch and shade the region $\mathcal{E}$ in $xyz$-space, and label your axes and any intercepts. Do not evaluate the integral.

$$\int_0^3 \int_0^y 4 \, dx \, dy$$

(b) Given

$$\int_0^{2} \int_0^{y/2} \int_5^{9-y^2} e^{3z} (y + 2x) \, dz \, dx \, dy$$

Rewrite as equivalent integral(s) using the ordering $dx \, dy \, dz$. Do not evaluate.