APPM 2350—Exam 3 (Chapter 12 and 13A)
Wednesday April 7th, 7:30pm-9pm 2021

Show all your work and simplify your answers. Answers with no justification will receive no points. .

Problem 1 (24 points)

Consider the integral $\displaystyle\iint_{\mathcal{R}} (x^2 + y^2)\ \mathrm{dA}$ where $\mathcal{R}$ is the region in the **first quadrant** given by

$$\mathcal{R} = \left\{ (x, y) \;\middle|\; 1 \le x \le \sqrt{2} \text{ and } x^2 + y^2 \le 4 \text{ and } y \ge 0 \right\}.$$

(a) Sketch and shade the region $\mathcal{R}$ in the first quadrant of the xy-plane. Label any intercepts.

(b) Set up the double integral(s) in terms of x and y with order of integration $dy\,dx$. Do not evaluate.

(c) Set up the double integral(s) in terms of x and y with order of integration $dx\,dy$. Do not evaluate.

(d) Set up the double integral(s) in terms of polar coordinates with order of integration $dr\,d\theta$. Do not evaluate.

(e) Which version of the double integral would you rather compute and, more importantly, **why**? (Just explain which one you would prefer to actually evaluate, but do not evaluate the integral).

Problem 2 (24 pts)

Use an appropriate uv-transformation to evaluate the integral

$$\iint_{\mathcal{R}} x e^{y - x^2}\, dA$$

where $\mathcal{R}$ is the region in the **first quadrant** bounded by $y = x^2$, $y = x^2 + 2$, $y = 2 - x^2$, and $y = 3 - x^2$.

Be sure to include a sketch of $\mathcal{R}$ and a sketch of the corresponding region S in the uv-plane in your solution.

Problem 3 (36 points)

Consider the following 2 spheres:

$$\text{Sphere A:} \quad x^2 + y^2 + z^2 = 2$$
$$\text{Sphere B:} \quad x^2 + y^2 + (z-1)^2 = 1$$

Let $\mathcal{E}$ be the intersection of the regions inside both spheres (i.e., the 3D solid that's common to both spheres).

Suppose the temperature at any point in space is given by $T(x, y, z) = x^2 y^2 z^2$ degrees Fahrenheit.

(a) Sketch and shade the cross section of $\mathcal{E}$ in the rz-plane. Label any intercepts.

(b) Sketch and shade the projection of $\mathcal{E}$ onto the xy-plane. Label any intercepts.

(c) Set up integral(s) to find the **volume of** $\mathcal{E}$ using spherical coordinates in the order $d\rho\,d\phi\,d\theta$. Do not evaluate the integral(s).

(d) Set up integral(s) to find the **z component of the centroid of** $\mathcal{E}$ using cylindrical coordinates in the order $dz\,dr\,d\theta$. Do not evaluate the integral(s).

(e) Set up integral(s) to find the **average temperature on the surface of the top hemisphere of Sphere A** (use the coordinate system that leads to the fewest number and simplest integrals). Do not evaluate the integral(s).

Problem 4 (16 points)

The following questions are not related:

(a) The following double integral gives the volume of a 3D region $\mathcal{E}$. Sketch and shade the region $\mathcal{E}$ in xyz-space, and label your axes and any intercepts. Do not evaluate the integral.

$$\int_0^3 \int_0^y 4 \; dx\, dy$$

(b) Given

$$\int_0^2 \int_0^{y/2} \int_5^{9-y^2} e^{3z}(y+2x)\, dz\, dx\, dy$$

Rewrite as equivalent integral(s) using the ordering $dx\, dy\, dz$. Do not evaluate.

End Of Exam