APPM 2350—Exam 2

Wednesday March 10th, 7:30pm-9pm 2021

Show all your work and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You must turn this in with your exam at the end. You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (28 pts)
Consider the function $f(x, y) = e^y(x^2 - y^2)$

(a) Find the quadratic (i.e. 2nd order) Taylor approximation of $f(x, y)$ centered at $(3, 0)$. (Tip: Double-check your work finding f_x and f_y before moving on, as the rest of the problem depends on these).
(b) Use Taylor's Formula to find a reasonable upper bound on the error of this quadratic approximation if $|x - 3| \leq 0.1$ and $|y| \leq 0.1$. (You can leave your answer unsimplified).
(c) Identify and classify all critical points of $f(x, y)$.

Problem 2 (18 points)
A beetle is walking along a path $\mathbf{r}(t)$ formed by the intersection of two rock surfaces. One surface, a curved vertical wall, is described by $y = \frac{1}{2}x^2$ and the other surface is a plane described by $z = 3x + \frac{1}{2}y$. (Ground level would be the xy-plane). The beetle starts at the origin and moves along the path towards the point P given by $(x, y, z) = (2, 2, 7)$. The temperature outside that morning is given by $T(x, y, z) = xyz + 20$.

(a) Determine a parameterization of the beetle's path, $\mathbf{r}(t)$, where t represents time.
(b) As the beetle passes over the point P, find the instantaneous rate of change of the temperature *with respect to time*, given your particular parameterization.
(c) As the beetle passes over the point P, calculate the instantaneous rate of change of the temperature with respect to *distance traveled* (in the direction of the beetle's path).

Problem 3 (18 pts) Ignoring air resistance, the height of a rocket t seconds after launch and with thrust $a\ ft/s^2$ can be modeled by $h(t, a) = \frac{1}{2}(a - 32)t^2$ feet. Limited fuel capacity imposes the constraint $a^2t = 10,000$, where a is once again the thrust and t the amount of time this thrust can be maintained before running out of fuel.

(a) Use the method of Lagrange multipliers to find the value of a that maximizes the height the rocket can reach. Simplify your answer.
(b) For the value of a you found in part (a), how long will the engines burn before running out of fuel? Leave your answer unsimplified.

Problem 4 (18 points)
Consider the function $g(x, y) = \frac{3x+2y}{\sqrt{x^2+y^2}}$

(a) Sketch the graph of the level curve of $g(x, y)$ that passes through the point $(-2, 3)$. Label axes and the value of g on the level curve.
(b) Prove algebraically that the following limit does not exist: $\lim_{(x,y)\to(0,0)} \frac{3x+2y}{\sqrt{x^2+y^2}}$

Problem 5 (18 pts) Let the height of a mountain be given by the function $H(x, y)$, with $H(1, 3) = 5,000$ ft and $\nabla H(1, 3) = 2\mathbf{i} - 7\mathbf{j}$

(a) You are standing on the mountain at the point $(1, 3, 5000)$ and you decide to hike in the steepest direction possible. Find a parametric equation for a line (in x, y and z) that is tangent to your path in this direction at the point $(1, 3, 5000)$
(b) Find the equation of the plane that's tangent to the mountain at the point $(x, y, z) = (1, 3, 5000)$. Give your answer in **standard form**, fully simplified.

End Of Exam