

APPM 2350—Exam 1 (Chapter 10)
Wednesday Feb 10th, 7:30pm-9pm 2021

Show all your work and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You must turn this in with your exam at the end. You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (24 points)

A particle travels along the curve of intersection of the surface $z = x^2 + 3y^2$ and the plane $y = 1$.

- Sketch and identify (give the name of) the surface $z = x^2 + 3y^2$. Label any intercepts and label axes.
- Give a position function, $\mathbf{r}(t)$, that traces out the particle's path.
- What is the curvature of this curve at the point $(\sqrt{6}, 1, 9)$? Fully simplify your answer.

Problem 2 (22 points)

Gravity causes a block to roll down a (frictionless) ramp. The block follows a path that is a straight line in the xz -plane with slope $-\frac{1}{3}$. Suppose the block has mass $m = 0.51$ kg so that the force of gravity is $\mathbf{F} = -5\mathbf{k}$ Newtons.

- How much work is done by gravity if the block travels a distance of 4 meters on the ramp?
- Find the force of gravity in the direction of the ramp (give your answer as a vector).

Problem 3 (32 points)

Two spacecraft are planning a mid-flight hand off. Their trajectories are given by

$$\text{Spacecraft 1: } \mathbf{r}_1(t) = 8\mathbf{i} + (3t - 5)\mathbf{j} + 2(t - 3)^2\mathbf{k}$$

and

$$\text{Spacecraft 2: } \mathbf{r}_2(t) = 4t\mathbf{i} + (t - 1)^2\mathbf{j} + 2\mathbf{k}$$

with distance measured in miles and time in minutes. To achieve this hand off, the technician will need to know the equation of the plane that contains the velocity vectors for both vessels at the time they pass each other.

- What are the (x, y, z) coordinates of the point where the two spacecraft meet?
- What is the equation of the plane tangent to both of the spacecraft's trajectories at the point where they meet? (i.e. what is the equation of the plane that contains each spacecraft's tangent line at the point where they meet?) Give your answer in **standard form**, fully simplified.
- At the instant they meet, the first spacecraft leaves its original path, flying straight ahead along a line at a constant speed of 3 miles/min. Give the (x, y, z) coordinates of Spacecraft 1 after it travels along this line for 10 minutes.

Problem 4 (22 points)

A few unrelated questions:

- Consider a particle whose position as a function of time t is given by

$$\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$$

How far does the particle travel **along the parameterized path** from time $t = 0$ to time $t = 3\pi/2$?

Tip: The identity $\sin^2\left(\frac{t}{2}\right) = \frac{1 - \cos t}{2}$ may be helpful.

- A bike travels around a circular track with radius 16m, at a speed that increases with time, given by $\|\mathbf{v}(t)\| = 3t^2$ m/s. Find the **magnitude** of the bike's acceleration at the instant when $t = 2$ seconds. (Tip: don't try to parameterize the bike's path during this exam - contact a professional later!)

End Of Exam
