INSTRUCTIONS: Write your name and your instructor's name on the front of your work. Work all problems. Show your work clearly. Note that a correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (36 points) Given

$$g(x, y) = -\sin(\pi xy) + 3\pi xy^2$$

   (a) Find the derivative of $g(x, y)$ at $(x, y) = (2, 1)$ in the direction toward the *point* $(x, y) = (3, -1)$.

   (b) In which direction(s) is the derivative of $g(x, y)$ at $(2, 1)$ equal to 0? (Give your answer(s) as unit vector(s)).

   (c) What is the *value* of the largest derivative of $g(x, y)$ at the point $(2, 1)$?

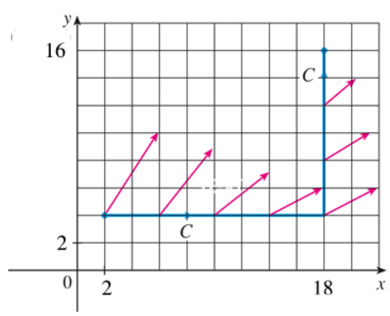
2. (14 points) The gravitational force on an object of mass m (located at the point (x, y, z)), due to another object of mass M (located at the origin), is given by the vector field:

$$\mathbf{F} = -GmM \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

   where G, m, and M are constants.

   Assume an object of mass M is located at the origin. Let P_1 and P_2 be points at a distance s_1 and s_2 (respectively) from the origin. The work done by the gravitational force field $\mathbf{F}$ as an object (of mass m) moves from P_1 to P_2 is independent of the path taken. Find this work. (Please leave your final answer in terms of s_1, s_2, G, m, and M). Fully justify your answer.

3. (12 points) Several vectors from the vector field $\mathbf{G}$ are shown below in pink. The path C, starts at $(x, y) = (2, 4)$ and ends at $(x, y) = (18, 16)$ as shown in blue in the figure. Use the figure below to *estimate* $\int_C \mathbf{G} \cdot \hat{\mathbf{T}}\, ds$ using the definition. (Provide the best estimate possible using *only the information given in the figure*). Fully justify your answer.



4. (36 points) Consider the vector field given by

$$\mathbf{F} = yz\mathbf{i} - y\mathbf{j} + z^2\mathbf{k}$$

   and let S denote the open surface that consists of the portion of the paraboloid described by $y = 10 - x^2 - z^2$ for $y \geq 1$.

   (a) Fully set-up (do not evaluate), integral(s) to find $\bar{y}$, the y-component of the centroid of the surface S. For full credit, set this up using the coordinate system that leads to the simplest and fewest number of integral(s).

   (b) Let $I = \iint\limits_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}}\, dS$, where $\hat{\mathbf{n}}$ is the unit normal to the surface S that points outward (i.e. away from the origin).

      i. Fully set-up (do not evaluate), *surface integral(s)* to directly evaluate I as defined above. For full credit, set this up using the coordinate system that leads to the simplest and fewest number of integral(s) and fully simplify the integrand(s).

      ii. Fully set-up (do not evaluate), a *path integral* that would also lead to the value of I. Clearly name any theorem(s) that you use here.

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5. (26 points) Given the plane

$$z = 8x - y$$

(a) Find the point on the plane that is closest to the point $(9, 4, 2)$.

(b) Find every point on the surface $2x^2 - y^2 + z^2 = 2$ where the tangent plane to the surface is parallel to the plane $z = 8x - y$.

6. (26 points) Let

$$\mathbf{F} = y\mathbf{i} - x\mathbf{j} + (4z^2 + 4)\mathbf{k}$$

Consider the finite object bounded on the top by the surface $z = 1$, on the bottom by $z = 0$, and on the sides by $x^2 + y^2 + z^2 = 4$.

(a) *Calculate* the outward flux of the vector field $\mathbf{F}$ over the entire surface of the object by separately calculating the outward flux through each side and adding together. Be sure to clearly identify the flux over each part of the bounding surface. **Pro Tip:** When evaluating, choose the coordinate system that leads to the simplest and fewest number of integrals.

(b) Verify your calculation in part(a) by re-doing the problem using a key theorem in Calculus III. State the theorem you use and *fully evaluate* any integral(s). **Pro Tip:** When evaluating, choose the coordinate system that leads to the simplest and fewest number of integrals.

ENJOY YOUR SUMMER AND STAY SAFE!