APPM 2350—Exam 3 (Chapter 12 and Scalar Line Integrals and Scalar Surface Integrals)
Wednesday April 15th, 7pm-8:30pm 2020

For each problem below, provide work justifying your full set-up of the limits of integration and fully simplify integrand(s) so all that is left to do is integrate. For full credit, set-up using the coordinate system that leads to the simplest integral(s).

Problem 1 (20 points) Consider a thin wire in the xy-plane that lies along the boundary of the region enclosed by the two curves

$$y = 2 - x \quad \text{and} \quad x = 4 - y^2 ,$$

where the units of x and y are length. The density along the wire, with units of mass per unit length, is given by

$$\delta(x, y) = \alpha x y^2 ,$$

where α is a known constant. Fully set-up integral(s) to find the total mass, M, of the wire. DO NOT EVALUATE your resulting integral(s).

Problem 2 (20 points) Consider the integral

$$\int_1^3 \int_1^{4-x} \left(\frac{x-y}{x+y} \right) dy\, dx .$$

Use the transformation

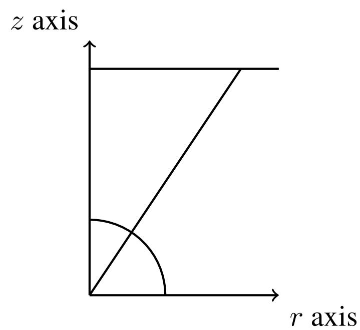
$$u = x - y \quad \text{and} \quad v = x + y$$

to fully set-up the equivalent integral(s) over an appropriate region in the uv-plane. DO NOT EVALUATE. For full credit, choose the set-up that results in the fewest possible number of integrals. **Pro tip**: For your own use (it won't be graded) you might consider drawing the region of integration in the xy-plane and in the new uv-plane.

Problem 3 (20 points) Let E be the solid region that consists of all points such that

$$x^2 + y^2 + z^2 \geq 4 \quad \text{AND} \quad z \leq 6 \quad \text{AND}\ z \geq \sqrt{3x^2 + 3y^2} .$$

To help you out, here is a sketch of the cross section of the surfaces of interest in an arbitrary θ plane. The figure is drawn to scale, but you will need to find the specific intersection locations, and most importantly, correctly identify the region of integration. **Pro tip**: Double check your intersections before proceeding with the problem! And then check them one more time.



The temperature in the solid is given by

$$T(x, y, z) = z(x^2 + y^2) .$$

Fully set up, BUT DO NOT EVALUATE, integral(s) to find

(a) The *average temperature* of the region using spherical coordinates in the order $d\rho, d\phi, d\theta$

(b) The *volume of the region* using cylindrical coordinates in the order $dr, dz, d\theta$

There are TWO more problems on the next page!

Problem 4 (20 points) Consider the hemispherical surface given by $x^2 + y^2 + z^2 = 9$ for $x \geq 0$. Set up integral(s) to find $\bar{x}$ (the x-component of the centroid) of the part of this hemispherical surface that lies inside the cylinder $y^2 + z^2 = 4$. For full credit, set this up using the coordinate system that leads to the simplest and fewest number of integral(s). **Pro tip**: sketch the object of interest, and reorient it and the axes as necessary, until you have it clear in your head what it looks like.

Problem 5 (20 points) Consider a cone of height H whose base has radius R. At the vertex of the cone (located at $x = 0$, $y = 0$, $z = H$) there is the source of a charge. At the base of the cone is a **circular metal disk** in the $z = 0$ plane. On that circular disk, the charge density at each point P is given by

$$\delta(x, y) = \frac{\alpha}{d(x, y)},$$

where $d(x, y)$ is the distance from P to the charge source location, and α is a known constant. Set up, but DO NOT EVALUATE integral(s), to find the **total charge on the circular metal disk**. (**Pro Tip:** Read that previous sentence again until you are confident about what you are integrating). For full credit, set up your calculation using the coordinate system, and integration order, that leads to the simplest integral(s).

End Of Exam
