INSTRUCTIONS:

Any work or answers you write on this sheet **will not** be graded. Please show all your work and provide all your answers in the **Exam 2 Solution Packet**, starting each problem on the correct page in the packet.

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1. (25 points) Consider the function $f(x,y) = y + \cos\left(\frac{\pi}{2}x\right) + \sin\left(\frac{\pi}{2}y\right)$.

   (a) Determine the **second order** Taylor approximation of $f(x,y)$ about the point $(-1,1)$.

   (b) Approximate the value of $f(-1.1, 1.01)$ using your second order approximation in part (a). Do not try to simplify your answer.

   (c) Find an upper bound for the magnitude of the error in a **linear approximation** assuming that you only use values of x and y such that $|x+1| \leq 0.1$ and $|y-1| \leq 0.01$. Read this again...error bound on a **linear approximation**. Again, do not try to simplify your result.

2. (25 points) Poisson Peak overlooks Lake Laplace. Locally, the elevation of the terrain is given, in kilometers above sea level, by the height function $h(x,y) = 3 + 3x - x^3 - 2y^2 + \frac{4}{3}y^3$.

   (a) Determine the location of the summit of Poisson Peak and its elevation above sea level. Write your final answer in the form (x,y,z).

   (b) Determine the location of the bottom of Lake Laplace and its elevation above sea level. Write your final answer in the form (x,y,z).

   (c) A stream emerges from a spot on the surface located at $(0, \frac{3}{2}, 3)$ and flows down the surface along the path of steepest descent. At the point $(0, \frac{3}{2}, 3)$, find a vector that is tangent to the stream and points in the direction the stream is flowing down the surface. Give your final answer as a vector with $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components.

3. (25 points) Some Calculus 3 space cadets are flying through space and collecting space dust to fuel their ship. The amount of space dust in this region is given by $D(x,y,z) = 2x + 4xy + 7z^2 + 16$. Their Collecto-matic sensor tells the instantaneous collection rate of space dust with respect to time, dD/dt.

   (a) The Cadets know the following information about their position and velocity at times $t = 0$ and $t = 2$:

   $$\mathbf{r}(0) = 0\,\mathbf{i} + 3\,\mathbf{j} + 2\,\mathbf{k} \qquad \mathbf{r}(2) = 2\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k} \qquad \mathbf{r}'(0) = 2\,\mathbf{i} - 5\,\mathbf{j} + \mathbf{k} \qquad \mathbf{r}'(2) = 6\pi\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\,.$$

   What does the Collecto-matic register for dD/dt when t is entered as they pass over the point $(0,3,2)$?

   (b) A young cadet accidentally swaps out a memory card, and now the ship's position is given by

   $$x(u,v,w) = e^{2u+v}, \quad y(u,v,w) = u\sin(w), \quad \text{and } z(u,v,w) = uw + v^2$$

   .

   If possible, determine what the Collecto-matic reads for dD/dt when $u = 3$, $v = -1$, and $w = \pi$. If this calculation is not possible, state what additional information is required.

4. (25 points) Consider the volume in the first octant formed by the three principal planes $x = 0$, $y = 0$, $z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where $a, b, c > 0$. A relatively easy calculation (as you will soon learn,) shows that the volume is $V = \frac{abc}{6}$. No, you don't have to do the volume calculation. In fact, don't even try it...you will only hurt yourself on this exam. Determine the values of a, b, and c, that minimize the volume if the plane must pass through the point $(3,2,1)$.