Write clearly and in the box:

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Read the following:

- **RIGHT NOW!** Write your name, student ID and section number on the top of your exam.
- You are allowed one 8.5 \times 11-in page of notes (ONE side). You must turn this in with your exam at the end.
- You may **NOT** use a calculator, smartphone, smartwatch or any other electronic device.
- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- If you need more space for answering a question, there are blank pages at the end of the exam. If you choose to use the extra pages, make sure to clearly indicate which problem you are continuing.
- You have **90 minutes** for this exam.
Problem 1: (20 points) Given the surface $9x^2 = y - z^2$

(a) (6 points) Give the official name of this surface and then sketch it in 3D. For full credit label any intercepts.

Name of Surface: ___________________________ Sketch of surface:

(b) (6 points) Let $C$ be the curve that is given by the intersection of this surface with the plane $y = 4$. Find a vector function for $C$.

(c) (8 points) Find parametric equations for the tangent line to the curve $C$ at the point $\left(\frac{1}{3}, 4, -\sqrt{3}\right)$.
Problem 2: (24 points) A particle travels along the path

$$\mathbf{r}(t) = \left\langle \frac{1}{2} \sin(2t), \frac{1}{2} \cos(2t), t\sqrt{8} \right\rangle, \quad 0 \leq t \leq \pi$$

(a) (8 points) How much further does the particle travel this way than if it had traveled directly between its starting and ending points along a straight path?
Problem 2 cont’d: A particle travels along the path

\[ \vec{r}(t) = \left< \frac{1}{2} \sin(2t), \frac{1}{2} \cos(2t), t\sqrt{8} \right>, \quad 0 \leq t \leq \pi \]

(b) (8 points) Find the unit tangent vector, \( \hat{T}(t) \) and the unit normal vector, \( \hat{N}(t) \)

(c) (8 points) For what values of \( t \in [0, \pi] \), if any, is the unit tangent vector \( \hat{T}(t) \) perpendicular to the plane \( \sqrt{3} x - y + 4\sqrt{2} z = 10 \)?
Problem 3: (24 points) A surveyor measures three points $P_1(1, 4, 1)$, $P_2(2, 4, 0)$, and $P_3(4, 0, 1)$, on a planar slope $S$.

(a) (8 points) Find an equation for the plane $S$, that passes through all 3 points. (Before moving on, check your final answer by plugging in all 3 points and verifying they are on the plane you found).

(b) (8 points) There is an object at the point $P_0(-1, 2, -16)$ that the surveyor needs to reach by drilling. Find the point on the plane $S$ that is closest to $P_0$ (so they can drill the shortest path). Give your final answer in the form $(x, y, z)$. 
Problem 3: (cont’d) A surveyor measures three points $P_1(1, 4, 1)$, $P_2(2, 4, 0)$, and $P_3(4, 0, 1)$, on a planar slope $S$.

(c) (8 points) They soon discover the ground along the shortest path is too hard to dig through. Instead, they determine that digging somewhere on the line that passes through the points $P_1$ and $P_2$ will be much easier on the equipment. Find the point on this line that is closest to $P_0(-1, 2, -16)$. Give your final answer in the form $(x, y, z)$. 
Problem 4: (32 points) Consider a particle moving along the parabolic curve \( x = ay^2 \) in the \( xy \) plane, where \( a > 0 \).

(a) (8 points) What is the curvature, \( \kappa \) at the point \( (x, y) = (a, 1) \)?

(b) (3 points) Is there a point on the parabola where \( \kappa \) has a maximum value? If so, what are the \( (x, y) \) coordinates and what is \( \kappa \) at this point? If not, explain why.

(c) (3 points) Is there a point on the parabola where \( \kappa \) has a minimum value? If so, what are the \( (x, y) \) coordinates and what is \( \kappa \) at this point? If not, explain why.
Problem 4: (cont’d) Consider a particle moving along the parabolic curve $x = ay^2$ in the $xy$ plane, where $a > 0$.

(d) (10 points) Suppose the particle travels along the entire curve with constant speed $||v|| = 5$ m/s. Write down the acceleration vector, $a$, of the particle when it is at the point $(x, y) = (a, 1)$.
Problem 4: (cont’d) Consider a particle moving along the parabolic curve $x = ay^2$ in the $xy$ plane, where $a > 0$.

(e) (8 points) Do you have enough information provided to determine the torsion, $\tau$, the particle is experiencing at the point $(a, 1)$? If so, find the torsion at this point. If not, explain what additional information you would need to determine it.
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