Write **clearly** and **in the box**:

Name:
Student ID:
Section number:

Read the following:

- **RIGHT NOW**! Write your name, student ID and section number on the top of your exam.
- You are allowed one 8.5×11-in page of notes (ONE side). You must turn this in with your exam at the end.
- You may **NOT** use a calculator, smartphone, smartwatch or any other electronic device.
- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- If you need more space for answering a question, there are blank pages at the end of the exam. If you choose to use the extra pages, make sure to **clearly** indicate which problem you are continuing.
- You have **90 minutes** for this exam.

Problem 1: (20 points) Given the surface $9x^2 = y - z^2$

(a) (6 points) Give the official name of this surface and then sketch it in 3D. For full credit *label any intercepts*.

Name of Surface: [ ] Sketch of surface:

(b) (6 points) Let C be the curve that is given by the intersection of this surface with the plane $y = 4$. Find a vector function for C.

(c) (8 points) Find parametric equations for the tangent line to the curve C at the point $(\frac{1}{3}, 4, -\sqrt{3})$

Problem 2: (24 points) A particle travels along the path

$$\vec{r}(t) = \left\langle \frac{1}{2}\sin(2t), \frac{1}{2}\cos(2t), t\sqrt{8} \right\rangle, \qquad 0 \le t \le \pi$$

(a) (8 points) How much further does the particle travel this way than if it had traveled directly between its starting and ending points along a straight path?

Problem 2 cont'd: A particle travels along the path

$$\vec{r}(t) = \left\langle \frac{1}{2}\sin(2t), \frac{1}{2}\cos(2t), t\sqrt{8} \right\rangle, \qquad 0 \le t \le \pi$$

(b) (8 points) Find the unit tangent vector, $\hat{\mathbf{T}}(t)$ and the unit normal vector, $\hat{\mathbf{N}}(t)$

(c) (8 points) For what values of $t \in [0, \pi]$, if any, is the unit tangent vector $\hat{\mathbf{T}}(t)$ perpendicular to the plane $\sqrt{3}\,x - y + 4\sqrt{2}\,z = 10$?

Problem 3: (24 points) A surveyor measures three points $P_1(1,4,1)$, $P_2(2,4,0)$, and $P_3(4,0,1)$, on a planar slope S.

(a) (8 points) Find an equation for the plane S, that passes through all 3 points. (Before moving on, check your final answer by plugging in all 3 points and verifying they are on the plane you found).

(b) (8 points) There is an object at the point $P_0(-1,2,-16)$ that the surveyor needs to reach by drilling. Find the point on the plane S that is closest to P_0 (so they can drill the shortest path). Give your final answer in the form (x,y,z).

Problem 3: (cont'd) A surveyor measures three points $P_1(1, 4, 1)$, $P_2(2, 4, 0)$, and $P_3(4, 0, 1)$, on a planar slope S.

(c) (8 points) They soon discover the ground along the shortest path is too hard to dig through. Instead, they determine that digging somewhere on the line that passes through the points P_1 and P_2 will be much easier on the equipment. Find the point on this line that is closest to $P_0(-1, 2, -16)$. Give your final answer in the form (x, y, z).

Problem 4: (32 points) Consider a particle moving along the parabolic curve $x = ay^2$ in the xy plane, where $a > 0$.

(a) (8 points) What is the curvature, κ at the point $(x, y) = (a, 1)$?

(b) (3 points) Is there a point on the parabola where κ has a maximum value? If so, what are the (x, y) coordinates and what is κ at this point? If not, explain why.

(c) (3 points) Is there a point on the parabola where κ has a minimum value? If so, what are the (x, y) coordinates and what is κ at this point? If not, explain why.

Problem 4: (cont'd) Consider a particle moving along the parabolic curve $x = ay^2$ in the xy plane, where $a > 0$.

(d) (10 points) Suppose the particle travels along the entire curve with constant speed $||\mathbf{v}|| = 5$ m/s. Write down the acceleration vector, $\mathbf{a}$, of the particle when it is at the point $(x, y) = (a, 1)$.

Problem 4: (cont'd) Consider a particle moving along the parabolic curve $x = ay^2$ in the xy plane, where $a > 0$.

(e) (8 points) Do you have enough information provided to determine the torsion, τ, the particle is experiencing at the point $(a, 1)$? If so, find the torsion at this point. If not, explain what additional information you would need to determine it.

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