On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 4 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or other electronic devices are permitted.

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1. [20 pts] Darth Vader is flying a TIE fighter along the path given by $\mathbf{r}(t) = \cos(\pi t)\,\mathbf{i} + \sin(\pi t)\,\mathbf{j} + (4t - t^2)\,\mathbf{k}$, $0 \le t \le 4$. During the flight, he fires a laser beam from the front of his ship that travels in a straight line. When $t = 3$, find the coordinates of the point where the laser beam penetrates the $xy-$plane.

2. [15 pts] Consider the equation $-\frac{1}{4}z^2 - 8y^2 + x^2 + 2z - 20 = 0$
   (a) [6 pts] Name the surface, providing justification for your answer.
   (b) [6 pts] Does the surface intersect the $yz-$plane? Justify your answer.
   (c) [3 pts] Name the conic section of the trace when $x = -\sqrt{32}$, providing justification for your answer.

3. [20 pts] Find the equation of the plane that is perpendicular to the plane $2z = 5x + 4y$ and contains the line with symmetric equations $-x = \dfrac{y+2}{5} = \dfrac{z-5}{-4}$. Write your final answer in the form $ax + by + cz = d$.

4. [45 pts] A scorpion is crawling on a shelf located 2 meters above the floor in a room. Its path is given by $\mathbf{r}(t) = \dfrac{t^3}{3}\mathbf{i} + \dfrac{t^2}{2}\mathbf{j} + 2\,\mathbf{k}, t > 0$, with distances measured in meters. Answer **ALL** of the following questions for $t = 1$ second.
   (a) [5 pts] Where is the scorpion?
   (b) [5 pts] How fast is the scorpion crawling? (Include units in your answer)
   (c) [10 pts] Briefly describe in words (*i.e.* DO NOT COMPUTE) what the following two quantities represent physically in terms of the scorpion's path:
   i. $\sqrt{\mathbf{r}(t) \cdot \mathbf{r}(t)}$ ii. $\displaystyle\int_0^t \sqrt{\mathbf{r}'(u) \cdot \mathbf{r}'(u)}\,\mathrm{d}u$
   (d) [5 pts] Find the scorpion's unit tangent vector, $\mathbf{T}$.
   (e) [5 pts] Find the unit normal to the scorpion's path by computing $\mathbf{T} \times \mathbf{k}$.
   (f) [5 pts] Find the binormal vector and the equation of the scorpion's osculating plane. Then, with the origin of an $xy-$coordinate system representing the scorpion's location, point your eyes in the direction of $\mathbf{B}$ and make a sketch of and label $\mathbf{T}$ and $\mathbf{N}$. You do not need to draw the actual path or location of the scorpion.
   (g) [5 pts] How fast is the scorpion's speed changing? (Include units in your answer)
   (h) [5 pts] Does the scorpion's acceleration possess a component normal to its path? If so, find it, including units in your answer. If not, explain why not.

PROJECTIONS, DISTANCES FROM POINT S TO LINE CONTAINING POINT P, AND S TO PLANE WITH NORMAL $\mathbf{n}$

$$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a} \qquad d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{\|\mathbf{n}\|}\right|$$

ARC LENGTH, FRENET FORMULAS, AND TANGENTIAL AND NORMAL ACCELERATION COMPONENTS

$$\mathrm{d}s = \|\mathbf{v}\|\,\mathrm{d}t \qquad \mathbf{T} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \qquad \mathbf{N} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}s}{\|\mathrm{d}\mathbf{T}/\mathrm{d}s\|} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}t}{\|\mathrm{d}\mathbf{T}/\mathrm{d}t\|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s} = \kappa\mathbf{N} \quad \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s} = -\tau\mathbf{N} \quad \kappa = \left\|\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s}\right\| = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|^3} = \frac{|f''(x)|}{\left\{1+[f'(x)]^2\right\}^{3/2}} = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \quad \tau = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad a_T = \frac{\mathrm{d}\|\mathbf{v}\|}{\mathrm{d}t} \qquad a_N = \kappa\|\mathbf{v}\|^2 = \sqrt{\|\mathbf{a}\|^2 - a_T^2}$$