- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/102324 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The function $g(x,y) = \begin{cases} \dfrac{\cos^2 x - \sin^2(y/2)}{\cos x - \sin(y/2)} & (x,y) \neq (0,\pi) \\ 0 & (x,y) = (0,\pi) \end{cases}$ is not continuous at $(0,\pi)$.

   (b) The linearization of $z = 2\cos x \sin y$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is $L(x,y) = 1 - x + y$

   (c) The level surface of the function $w(x,y,z) = 3x^2 + y^2 + z^2$ corresponding to $w = -2$ is an ellipsoid.

   (d) If $t = u/v$ and $u = x^2 - y^2$ and $v = 4xy^3$, then $\partial t/\partial x = \left(x^2 + y^2\right)/(4x^2y^3)$.

   (e) There exists a unit vector $\mathbf{u}$ such that the rate of change of $f(x,y) = x^3 + e^y$ in the direction of $\mathbf{u}$ at $(x,y) = (1,0)$ is 10.

2. [2350/102324 (16 pts)] You are provided the following information about the function $f(x,y)$. Use it to answer the questions below.

$$\begin{array}{llll} f(0,-1) = 0 & f_x(0,-1) = 0 & f_y(0,-1) = 0 & \\ f_{xx}(0,-1) = 1 & f_{xy}(0,-1) = -2 & f_{yy}(0,-1) = 0 & \\ f_{xxx}(x,y) = 1 & f_{xxy}(x,y) = 5x^2 & f_{xyy}(x,y) = -\dfrac{7}{y} & f_{yyy}(x,y) = 0 \end{array}$$

   (a) (6 pts) Find an appropriate quadratic Taylor polynomial for $f(x,y)$.

   (b) (4 pts) Estimate $f(0.1, -0.8)$.

   (c) (6 pts) Find an upper bound on the error in the approximation in part (b) if $|x| \leq 0.2$ and $|y+1| \leq 0.3$.

3. [2350/102324 (20 pts)] Consider the function $w(x,y,z) = 3z^2 + e^{3z+3} + \ln\left(\frac{1}{2}xy\right)$.

   (a) [4 pts] What is the domain of w?

   (b) [6 pts] Find the equation of the tangent plane to the level surface $w = 3$ at the point $(1,2,-1)$. Write your answer using the standard form $ax + by + cz = d$.

   (c) [10 pts] Now suppose you are traveling along the path $\mathbf{r}(t) = e^t\,\mathbf{i} + (3 - \cos t)\,\mathbf{j} + \left(t^2 + t - 1\right)\mathbf{k}$ where t represents time (minutes), $w(x,y,z)$ gives the amount (grams) of moisture in the air and distances are in km. Answer the following, assuming you are at the point $(x,y,z) = (1,2,-1)$. Include appropriate units.

      i. With respect to time, is the moisture increasing, decreasing or not changing? If increasing or decreasing, at what rate?

      ii. Find the rate of change of moisture with respect to distance.

MORE PROBLEMS BELOW/ON REVERSE

4. [2350/102324 (34 pts)] You need to find the dimensions that minimize the total cost of material needed to construct a rectangular box having the following characteristics: (i) it is open-topped (has five sides); (ii) its volume is 600 cubic inches; (iii) the material for its bottom costs 6¢ per square inch; (iv) the material for the sides costs 5¢ per square inch; (v) l, w, h are the box's length, width and height, respectively. Perform the following steps to do this problem. Do not use Lagrange Multipliers and include units in your answer.

   (a) [8 pts] Show that the cost function can be written as $C(l, w) = 6000\left(l^{-1} + w^{-1}\right) + 6lw$.

   (b) [12 pts] Find l, w that minimize the cost function.

   (c) [8 pts] Verify that the l, w you found in part (b) do result in a minimum of the cost function.

   (d) [3 pts] What are the dimensions of the cheapest box?

   (e) [3 pts] How much will it cost to build the box?

5. [2350/102324 (20 pts)] The satisfaction you get from eating x slices of bacon, y eggs and z potatoes is given by the satisfaction function $s(x, y, z) = \sqrt{xyz}$. Suppose you have six dollars to spend and a rather hearty appetite. If each slice of bacon costs \$0.50, each egg costs \$0.25, and each potato costs \$0.10, use Lagrange Multipliers to determine how many bacon slices, eggs and potatoes you should purchase to be the most satisfied.