- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED <u>AND</u> UPLOADED YOUR EXAM TO GRADESCOPE.
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5" × 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.
- 0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.
- 1. [2350/092524 (10 pts)] Write the word TRUE or FALSE as appropriate. No work need be shown. No partial credit given.
 - (a) For any path $\mathbf{r}(t)$, the acceleration, $\mathbf{a}(t)$, and the binormal, $\mathbf{B}(t)$, are always orthogonal.
 - (b) For nonzero vectors A and B, $(A \times B) \cdot (A + B) = 0$.
 - (c) $\operatorname{comp}_{i}(2 \mathbf{j} + \mathbf{k}) = 1.$
 - (d) $2x^2 + 12x = y^2 4z 18$ is an elliptic paraboloid.
 - (e) The equation of the osculating plane of the path $\mathbf{r}(t) = e^t \mathbf{j} + \ln t \mathbf{k}, t > 0$, is x = 0.
- 2. [2350/092524 (24 pts)] Consider the points P(-1, 5, 3) and Q(6, 2, -2).
 - (a) (8 pts) Find an equation for the set of all points that are equidistant from P and Q. Simplify your answer completely and name the surface.
 - (b) (8 pts) Find the work done by $\mathbf{F} = 5 \mathbf{i} 3 \mathbf{j} + 7 \mathbf{k}$ moving an object from P to Q.
 - (c) (8 pts) Suppose an unknown force, **F**, applied at the point Q and normal to \overrightarrow{PQ} , produces a torque of magnitude $9\sqrt{83}$ about point P. What is the magnitude of the force?
- 3. [2350/092524 (22 pts)] You are playing indoors with a paper airplane, the velocity of which is given by $\mathbf{v}(t) = (1 3\cos t)\mathbf{i} + 3\sin t\mathbf{k}$.
 - (a) [6 pts] Find the tangential component of the acceleration vector.
 - (b) [10 pts] If the initial position of the airplane is $\mathbf{r}(0) = \mathbf{i} \mathbf{j} + \mathbf{k}$, find an expression for the path of the airplane.
 - (c) [6 pts] Look out! On its maiden voyage the airplane crashes into the ceiling, considered as a plane parallel to the xy-plane. The crash occurs when $t = \pi/2$. Find the acute angle at which the airplane hits the ceiling.
- 4. [2350/092524 (20 pts)] A mosquito is buzzing along the path $\mathbf{r}(t) = t \mathbf{i} + \frac{2\sqrt{2}}{3}t^{3/2}\mathbf{j} + \frac{1}{2}t^2\mathbf{k}, t \ge 0$, with distance measured in yards and time in seconds.
 - (a) [10 pts] Starting from time t = 0, how long does it take the mosquito to travel 12 yards along this path?
 - (b) [5 pts] After 1 second, how far is the mosquito from its starting position at t = 0?
 - (c) [5 pts] Now suppose that after flying along the aforementioned path for 2 seconds, the mosquito notices a bat approaching from behind. In an effort to avoid becoming the bat's dinner, from this point the mosquito leaves the original path, flying straight ahead along a line at a constant speed of 1 yard/sec. Find the mosquito's coordinates after it travels along this line for 3 seconds.
- 5. [2350/092524 (24 pts)] A bee leaves its hive located in a tree at the point A(1, 2, 3) and flies in a straight line to a sunflower located at point B(-1, 4, 7). From there it flies in a straight line to a daisy at point C(2, -5, -1) and returns on a straight path back to the hive.
 - (a) (8 pts) Find the area of the triangular region inside the bee's path.
 - (b) (8 pts) Find the equation of the plane containing the bee's path. Write your answer in the form ax + by + cz = d.
 - (c) (8 pts) The queen bee leaves the hive and crawls along a straight branch to point D at the origin. Find the volume of the parallelepiped formed by the queen bee's path, and the bee's path from the hive to the sunflower and from the hive to the daisy.