- This exam is worth 150 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on both sides.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/121923 (24 pts)] Write the word **TRUE** or **FALSE** as appropriate. Write your answers in a single column separate from any work you do. No work need be shown. No partial credit given.

   (a) If $\mathbf{a} = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\,\mathbf{k}$ and $\mathbf{c} = -3\,\mathbf{j} + \mathbf{k}$, then $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 1$.

   (b) The first order Taylor polynomial for $h(x, y) = e^{-x^2-y^2}$ centered at $(0, 0)$ is $T_1(x, y) = 1$.

   (c) The curvature of the path $\mathbf{r}(s)$ given by the arc length parameterization $\mathbf{r}(s) = \dfrac{\sqrt{3}}{2}s\,\mathbf{i} + \sin\dfrac{s}{2}\,\mathbf{j} - \cos\dfrac{s}{2}\,\mathbf{k}$ is always $\kappa = 0.25$.

   (d) If $f(x, y, z) = \ln\left(x^2 + y^3 + z^4\right)$, there exists a unit vector $\mathbf{w}$ such that the instantaneous rate of change of f with respect to distance in the direction of $\mathbf{w}$ at the point $(1, -1, 1)$ equals 10.

   (e) The equations of the normal line and tangent plane to the surface $x^2 + y^2 - z^2 = 4$ at the point $P(2, 1, -1)$ are, respectively, $\mathbf{l}(t) = \langle 4t + 2, 2t + 1, 2t - 1\rangle$, t a real number, and $2x + y + z = 4$.

   (f) Particles moving along straight line paths always experience zero acceleration.

2. [2350/121923 (20 pts)] You are going on a trip and want to maximize the space in your carry-on suitcase, assumed to be in the shape of a rectangular box. Find the dimensions of the suitcase of maximum volume if the sum of the suitcase's width (w), height (h) and two times its length (l) is 120 cm. No credit for using Lagrange Multipliers. Instead, solve this as a two dimensional optimization problem, verifying your result using the Second Derivatives Test.

3. [2350/121923 (25 pts)] The Hundred Acre Wood is under the influence of the vector field $\mathbf{F} = e^y\,\mathbf{i} + (xe^y + \sin z)\,\mathbf{j} + y\cos z\,\mathbf{k}$. Consider the points $M(0, 1, 0)$ and $N(\pi, 3, 2\pi)$ in the forest.

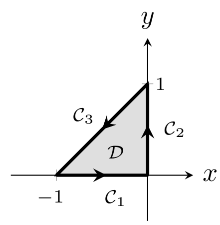
   (a) (8 pts) Winnie the Pooh waddles along the path $\mathbf{r}(t) = \left[\frac{\pi}{2}(t-1)\right]\mathbf{i} + t\,\mathbf{j} + \pi\,(t-1)\,\mathbf{k}$ between the points. Set up and simplify, but **do not evaluate**, the integral that will directly compute the amount of work Pooh does in waddling along the path from point M to point N. Pooh's friends Kanga and Roo offer to help you with the simplification by telling you that $\sin[\pi(t-1)] = -\sin\pi t$ and $\cos[\pi(t-1)] = -\cos\pi t$.

   (b) (14 pts) Pooh's gloomy friend Eeyore watches as Tigger bounces along the path $(t - \sin 4t)\,\mathbf{i} + (2 - \cos 3t)\,\mathbf{j} + 2t\,\mathbf{k}$, $0 \le t \le \pi$ and Owl flies along the path $\sqrt{\frac{t^3}{8\pi}}\,\mathbf{i} + \left(1 + \frac{t}{\pi}e^{\pi - t/2}\right)\mathbf{j} + t\,\mathbf{k}$, $0 \le t \le 2\pi$. Both of these paths begin at point M and end at point N. Eeyore claims that both Tigger and Owl did the same amount of work, W, as Pooh did and announces that an important Calculus 3 theorem can be used to compute W. Show that Eeyore is correct and use the theorem to find W. Hint: No difficult integration is required.

   (c) (3 pts) Finally, Pooh's other acquaintance, Gopher, crawls from point N to the point $(2, 3, 50)$ then tunnels underground back to point N. How much work did he do?

4. [2350/121923 (20 pts)] Use Stokes' Theorem to find the circulation of $\mathbf{V} = -2yz\,\mathbf{j} + 6yz\,\mathbf{k}$ around $\mathcal{C}$, the intersection of the plane $x + 2y + 6z = 1$ with the first octant, oriented clockwise when viewed from above.

5. [2350/121923 (33 pts)] Consider the vector field $\mathbf{F}(x, y) = x^2 y\,\mathbf{i} + xy^2\,\mathbf{j}$ and the following figure. As shown below, $\mathcal{D}$ has oriented boundary $\partial\mathcal{D} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3$.



(a) (6 pts) Compute $\mathbf{k} \cdot \nabla \times \mathbf{F}$ and determine if this function attains an absolute maximum and minimum value on the region $\mathcal{D} \cup \partial\mathcal{D}$. Do not find these values, if they exist, simply justify your answer in a few words.

(b) (3 pts) Without doing any integration, explain in words why there is no flux through or flow along $\mathcal{C}_1$ and $\mathcal{C}_2$.

(c) (10 pts) Using the parameterization $x = -t, y = -t + 1$, evaluate $\displaystyle\int_{\mathcal{C}_3} P\,\mathrm{d}y - Q\,\mathrm{d}x$.

(d) (4 pts) Using your answers to parts (b) and (c), what is $\displaystyle\int_{\partial\mathcal{D}} \mathbf{F} \cdot \mathbf{n}\,\mathrm{d}s$? Hint: no integration is required.

(e) (10 pts) Find the flux through $\partial\mathcal{D}$ using Green's Theorem.

6. [2350/121923 (28 pts)] Consider the surface $z = x^2 + y^2$ and vector field $\mathbf{E} = x\,\mathbf{i} + y\,\mathbf{j} + 2z\left(x^2 + y^2\right)\mathbf{k}$.

(a) (3 pts) Name the surface.

(b) (25 pts) Let $\mathcal{S}$ be that portion of $z = x^2 + y^2$ with $0 \le z \le 1$ and let $\mathcal{S}_1$ be the disk $x^2 + y^2 \le 1$ lying in the plane $z = 1$. Then $\mathcal{S} \cup \mathcal{S}_1$ is a closed surface enclosing the solid region $\mathcal{W}$.

i. (10 pts) Compute the upward flux of $\mathbf{E}$ through $\mathcal{S}_1$.

ii. (10 pts) Compute $\displaystyle\iiint_{\mathcal{W}} \nabla \cdot \mathbf{E}\,\mathrm{d}V$.

iii. (5 pts) Use Gauss' Divergence Theorem to find the downward flux of $\mathbf{E}$ through $\mathcal{S}$, which is not a closed surface. Hint: no integration is necessary.