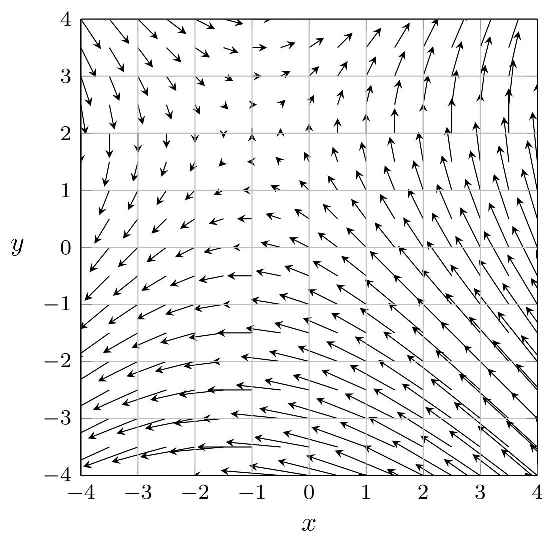
- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/112923 (15 pts)] Write the word **TRUE** or **FALSE** as appropriate. Write your answers in a single column separate from any work you do. No work need be shown. No partial credit given.

   (a) The area of one side of a fence built on the curve $y = x^2$ in the $z = 0$ plane for $0 \le x \le 3$ with height $z = f(x, y) = 1 + 4y$ is $\displaystyle\int_0^3 (1 + 4t^2)\,\mathrm{d}t$.

   (b) The vector field $\mathbf{V} = (y-2)\,\mathbf{i} + (x+1)\,\mathbf{j}$ is shown in the accompanying figure.



   (c) Any vector field of the form $f(y,z)\,\mathbf{i} + g(x,z)\,\mathbf{j} + h(x,y)\,\mathbf{k}$, where the appropriate partial derivatives of f, g, h exist, is incompressible.

   (d) For any vector field $\mathbf{F}$, $\nabla \cdot (\nabla \times \mathbf{F}) = \nabla \times (\nabla \cdot \mathbf{F})$.

   (e) If a vector field $\mathbf{V}$ has only $\mathbf{i}$- and $\mathbf{k}$-components, both of which are functions of only x and z and whose partial derivatives are nonzero, then the curl of $\mathbf{V}$ will have only a $\mathbf{j}$-component.

2. [2350/112923 (16 pts)] You need to compute $G = \displaystyle\int_{\mathcal{R}} \frac{x^2 - y^2}{x^2 + y^2}\,\mathrm{d}A$, where $\mathcal{R}$ is region in the first quadrant bounded by the curves

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 8, \quad x^2 - y^2 = 2, \quad x^2 - y^2 = -2$$

   (a) (2 pts) Describe in words what the quantity G represents.

   (b) (14 pts) Use the change of variables $u = \frac{1}{2}\left(x^2 + y^2\right)$ and $v = \frac{1}{2}\left(x^2 - y^2\right)$ to set up, **but not evaluate**, an appropriate integral to compute G.

3. [2350/112923 (25 pts)] Winnie the Pooh has his eyes on a honey-filled beehive in the shape of $4z = 4 - x^2 - y^2$. He can see the portion of the hive between the planes $y = x$ and $y = -x$ where $y \ge 0$. The surface of the hive is covered with bees and his friend Rabbit tells him that the density of the bees is $\delta(x,y,z) = 100y^2/\sqrt{x^2+y^2+4}$ bees per square centimeter. He also reminds him that $\sin^2 x = (1 - \cos 2x)/2$. How many bees does Pooh see? (it won't be a whole number, but that's ok)

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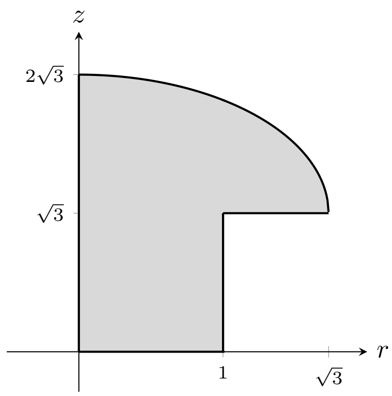
4. [2350/112923 (24 pts)] A constant θ slice through a delicious red velvet cupcake is shown in the following figure. The top of the cupcake is a portion of $x^2 + y^2 + \left(z - \sqrt{3}\right)^2 = 3$ and the mass density of the cupcake is $\delta(x, y, z) = y^2/(z + 1)$. Set up, **do not evaluate**, integral(s) to compute the following, using the given integration order.



(a) (4 pts) The mass of the portion of the cupcake above the plane $z = \sqrt{3}$, $\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$.

(b) (8 pts) The mass of the entire cupcake, $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$.

(c) (12 pts) The mass of the entire cupcake, $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$.

5. [2350/112923 (20 pts)] A thin triangular plate (lamina) lying in the xy-plane with density $\rho(x, y) = 2y$ is bounded by the lines $y = 1$, $y = 2 - x$ and $y = 2 + x$. The mass of the lamina is $m = 8/3$.

(a) (8 pts) Find the moment of inertia about the x-axis, integrating with respect to x first.

(b) (8 pts) Find the moment of inertia about the y-axis, integrating with respect to y first.

(c) (4 pts) Find the radii of gyration with respect to the x- and y-axes.