- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”× 11” crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: ”I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/102523 (15 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The function $g(x,y) = \begin{cases} \dfrac{1+xy}{y^2-x^2} & (x,y) \neq (-1,1) \\ 0 & (x,y) = (-1,1) \end{cases}$ is continuous at $(x,y) = (-1,1)$.

   (b) The function $f(x,y) = e^{-x-y}\cos\left(\pi x^2 y^3\right)$ is guaranteed to have a maximum and minimum value on the triangular region bounded by $x \geq 0, y \geq 0, x+y < 1$.

   (c) If $x(u,v) = 6\left(\sqrt{u}+1\right) - v^2, u = rst$ and $v = \dfrac{rs}{t}$, $\dfrac{\partial x}{\partial t} = 3\sqrt{\dfrac{rs}{t}} + 2\dfrac{r^2 s^2}{t^3}$.

   (d) If $h(x,y)$ is a function whose partial derivatives of all orders are continuous throughout $\mathbb{R}^2$ and if $h_x(2,3) = h_y(2,3) = 0$, then $h(2,3)$ must be a local extreme value of $h(x,y)$.

   (e) The equation of the tangent plane to the surface $(x-1)^2 + (y+2)^2 - z^2 = -16$ at the point $(-1,2,6)$ is $x - 2y + 3z = 13$.

2. [2350/102523 (17 pts)] You are wandering around on the surface $xy - z^2 = 1$. Use Lagrange Multipliers to determine the closest that you will get to the origin.

3. [2350/102523 (17 pts)] You are told that for a given function $f(x,y)$, its gradient is $\nabla f(x,y) = \left(3x^2+3y\right)\mathbf{i} + \left(3y^2+3x\right)\mathbf{j}$. Find and classify the critical points of $f(x,y)$.

4. [2350/102523 (31 pts)] Winnie the Pooh is on another quest for honey in the Hundred Acre Wood. The elevation of the ground in the woods is given by the function $h(x,y) = \ln xy$.

   (a) (4 pts) Find the domain of the elevation function, $h(x,y)$.

   (b) (7 pts) Find the slope of the ground in the direction of $2\,\mathbf{i} + \mathbf{j}$ at the point $(1,1,0)$.

   (c) (10 pts) Pooh is walking towards a beehive full of honey, following the path in the xy-plane given by $\mathbf{r}(t) = e^t\,\mathbf{i} + e^{2t}\,\mathbf{j}$. As he passes through the point $\left(e, e^2\right)$, how fast is his elevation changing with respect to time? What is his elevation there?

   (d) (10 pts) Pooh’s friend Piglet is standing guard at the honey-filled beehive located at the point $(3,9,3\ln 3)$. Those pesky bees note that something is amiss with their honey and come after Piglet. In what direction should Piglet start running in order to begin losing elevation the fastest and escape the bees? Write your answer as a vector in the xy-plane. What is Piglet’s instantaneous rate of change of elevation?

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5. [2350/102523 (20 pts)] The following problems are not related.

(a) (10 pts) According to the ideal gas law, the pressure (P), temperature (T) and volume (V) of a confined gas are related by $P = RT/V$ where R is a constant. Use differentials to approximate the percentage change in pressure $(\mathrm{d}P/P)$ if the temperature of a gas is increased 3% and the volume is increased by 5%.

(b) (10 pts) Your friends have found the first order Taylor polynomial for the function $f(x, y)$, centered at $(1, -1)$. They want to use this polynomial to approximate $f(x, y)$ when x and y satisfy the inequalities $|x - 1| \le 0.1$ and $|y + 1| \le 0.2$ and need to know about the error in the approximation. They have been kind enough to provide you with the following derivative information,

$$f_{xx}(x, y) = \frac{-4}{(1 + 2x - 4y)^2}, \quad f_{xy}(x, y) = \frac{8}{(1 + 2x - 4y)^2}, \quad f_{yy}(x, y) = \frac{-16}{(1 + 2x - 4y)^2},$$

as well as a graph of the level curves of $1/(1+2x-4y)^2$ shown in the figure. Based on the given information, what is the largest error your friends can expect the approximation to contain?