- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/092723 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) If $\mathbf{a}$ is a unit vector and $\mathbf{b} = 3\mathbf{a}$, then $\mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{b} \times \mathbf{a} = 3\mathbf{b}$.

   (b) For any smooth path $\mathbf{r}(t)$, the magnitude of $\mathbf{T} \times \mathbf{B} \times \mathbf{N}$ is zero.

   (c) Consider the arbitrary, non-zero, vectors $\mathbf{U}$ and $\mathbf{V}$. The projection of $\mathbf{U} \times \mathbf{V}$ onto $\mathbf{U}$ is the zero vector, $\mathbf{0}$.

   (d) The work done moving 6 units in the direction of $\mathbf{a} = 6\,\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$ subject to the force $\mathbf{F} = 2\,\mathbf{i} + 7\mathbf{j} + 3\,\mathbf{k}$ Nt is 35 Nt-m.

   (e) Suppose $\mathbf{A}$ and $\mathbf{B}$ are arbitrary, nonzero vectors with equal magnitude. Then $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$ are orthogonal.

2. [2350/092723 (30 pts)] Winnie the Pooh is on an adventure to track down some of his beloved honey. He spies a beehive high up in a tree and his friend Christopher Robin has given him some balloons that will lift him up to the hive. When $t = 0$, Christopher Robin releases Pooh Bear from the point $(0, 1, 0)$ and he floats along the path $\mathbf{r}(t) = \sin 4t\,\mathbf{i} + \cos 4t\,\mathbf{j} + 2t\,\mathbf{k}$ until $t = 2\pi$, at which time he arrives at the beehive.

   (a) (8 pts) How far did Pooh travel?

   (b) (6 pts) To get a better view of Pooh when he is at the beehive enjoying the honey, Christopher walks into a small pit located at $(5, 6, -\pi)$. How far is he from the beehive?

   (c) (8 pts) As Pooh is indulging himself, a gust of wind in the direction $3\,\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$ hits when $u = 0$ and blows him from the beehive in a straight line for $u > 0$. Find the parametric equations of this line, using u as the parameter.

   (d) (8 pts) A swarm of bees, to whom the honey belongs, is flying along the path $\mathbf{r}_b(u) = (6 + u)\,\mathbf{i} + \left(4 - u^2\right)\mathbf{j} + \left(\sqrt{3u} + 4\pi\right)\mathbf{k}$. They are irritated that Pooh has stolen their honey and are out to get him. Where do the bees meet up with Pooh? [Note that this is the same u as in part (c)].

3. [2350/092723 (17 pts)] The following problems are not related.

   (a) (8 pts) A wrench 30 cm long lies along the positive y-axis and grips a bolt at the origin. A force is applied in the direction of $\langle 0, 3, -4\rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 100 Nt-m of torque to the bolt.

   (b) (9 pts) Consider the equation $-x^2 + 4x + 3z^2 + ay^2 + 2y + 2 = 0$. Determine the quadric surface that results when $a = -1, 0, 1$.

4. [2350/092723 (17 pts)] The following problems are not related.

   (a) (9 pts) Consider the vectors $\mathbf{u} = 2\,\mathbf{i} - 3\,\mathbf{j}$, $\mathbf{v} = t\,\mathbf{i} + 3\,\mathbf{k}$, and $\mathbf{w} = -3\,\mathbf{i} + 2\,\mathbf{j} + t\,\mathbf{k}$. For what value(s) of t will the parallelepiped formed by the three vectors have a volume of 24 units?

   (b) (8 pts) A 100-meter dash is run on a track in the direction of the vector $\mathbf{a} = 5\,\mathbf{i} + 12\,\mathbf{j}$. The wind velocity $\mathbf{w} = 7\,\mathbf{i} + 2\,\mathbf{j}$ km/hr. The rules say that the wind speed in the direction of the race must not exceed 5 km/hr. Will the race results be disqualified due to an excessive wind? Justify your answer using Calculus 3 concepts.

5. [2350/092723 (26 pts)] A comet is flying through space along the path given by $\mathbf{r}(t) = \left(\frac{t^2}{2} + t\right)\mathbf{i} + \frac{t^2}{2}\,\mathbf{j} + (t - 1)\,\mathbf{k}$ where t (time) is a real number. Hint: You do not have to find $\mathbf{T}$ or $\mathbf{N}$ to do this problem.

   (a) (8 pts) Find the point on the path where the comet's speed is not changing.

   (b) (8 pts) Is the direction of the comet always changing? Justify your answer mathematically.

   (c) (10 pts) Find the equation of the normal plane (formed by the normal and binormal vectors) when the normal plane is parallel to the plane $x + 2y - z = 1$. Write your answer in the form $ax + by + cz = d$.