- This exam is worth 150 points and has 6 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on two sides.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/121222 (21 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The error in the quadratic Taylor polynomial for a function $g(x, y)$ is related to g_{xx}, g_{xy} and g_{yy}.

   (b) The traces in all planes parallel to all coordinate planes of the function $2z + 5x^2 - 3y^2 = 0$ are parabolas.

   (c) The principal unit normal vector, $\mathbf{N}$, for the helical path $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$, $t \geq 0$ is always parallel to the xy-plane.

   (d) The line of intersection of the planes $\frac{1}{2}x + y + 4z = 10$ and $x + 2y + 8z = -10$ has direction vector $\langle 8, 4, 1 \rangle$.

   (e) The three nonzero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, all lie in the same plane if $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$.

   (f) The Cartesian/rectangular $(z = -\sqrt{x^2 + y^2})$, cylindrical $(z = -r)$ and spherical $(\phi = 3\pi/4)$ equations represent the same surface.

   (g) An object moving with velocity $\mathbf{v}$ on a circle of radius 4 experiences a larger normal component of acceleration than one traveling at the same velocity on a circle of radius 1/4.

2. [2350/121222 (30 pts)] Let $\mathbf{V} = \left(x^2 + 4xy\right)\mathbf{i} - 6y\,\mathbf{j}$ be a velocity vector field.

   (a) [15 pts] Find the flow of $\mathbf{V}$ along the curve $\mathcal{C}$ given by $y = x^2 - x$ for $0 \leq x \leq 1$.

   (b) [15 pts] Find the total outward flux of $\mathbf{V}$, as a function of a and b, across the boundary of the rectangular region $0 \leq x \leq a,\ 0 \leq y \leq b$ where a, b are positive constants.

3. [2350/121222 (20 pts)] Let $\mathbf{F} = (e^x \sin y - yz)\,\mathbf{i} + (e^x \cos y - xz)\,\mathbf{j} + (z - xy)\ \mathbf{k}$.

   (a) [6 pts] Show that $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r} = 0$ for all closed paths in $\mathbb{R}^3$.

   (b) [8 pts] Find the potential function $f(x, y, z)$ for the field $\mathbf{F}$.

   (c) [6 pts] Calculate the work done by the force $\mathbf{F}$ moving an object along the line segment, $\mathcal{C}$, from $(0, \pi/2, -1)$ to $(1, \pi, 2)$.

4. [2350/121222 (22 pts)] Consider the vector field $\mathbf{F} = -3y\,\mathbf{i} + 3x\,\mathbf{j} + z^2\,\mathbf{k}$. Let $\mathcal{S}$ be that portion of the surface $3x^2 + 3y^2 - z^2 + 1 = 0$ lying above the plane $z = -\sqrt{13}$ and below the xy-plane.

   (a) [2 pts] Name the surface.

   (b) [10 pts] Compute the downward flux of the curl of $\mathbf{F}$ through $\mathcal{S}$ by directly evaluating $\displaystyle\iint_{\mathcal{S}} \boldsymbol{\nabla} \times \mathbf{F} \cdot \mathrm{d}\mathbf{S}$.

   (c) [10 pts] Compute the quantity in part (b) by evaluating an appropriate line integral.

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5. [2350/121222 (27 pts)] Let $\mathcal{S}$ be the portion of the surface $x^2 + y^2 + 2z = 16$ between the planes $z = 0$ and $z = 6$. Let $\mathbf{E} = \langle y, -x, 2z \rangle$ represent an electric field.

   (a) (2 pts) $\mathcal{S}$ is a portion of what quadric surface?

   (b) (10 pts) Compute the upward flux of the electric field through the surface by evaluating $\displaystyle\iint_{\mathcal{S}} \mathbf{E} \cdot \mathbf{n} \, \mathrm{d}S$ directly.

   (c) (15 pts) The surface $\mathcal{S}$ is not closed. However, Gauss' Divergence Theorem can still be applied to find the flux of $\mathbf{E}$ through $\mathcal{S}$. Apply the theorem to verify your answer to part (b).

6. [2350/121222 (30 pts)] A square metal plate occupies the region $0 \le x \le 20, 0 \le y \le 20$. The temperature of the plate is given by $T(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$. In all of the parts below, be sure to use Calculus 3 concepts. No credit for reducing the problems to a single variable problem.

   (a) [8 pts] An ant is crawling around on the plate, following the curve $y = 1 + x^2$. At the point $(1, 2)$ in the direction of the ant's motion, how is the temperature changing with respect to

      i. distance?

      ii. time?

   (b) [10 pts] On this path, will the ant ever experience the warmest temperature in the plate's interior? Fully justify your answer.

   (c) [12 pts] Now suppose the ant is crawling along the portion of $x + y = 12$ lying on the plate. Find the highest and lowest temperatures the ant experiences on this new path.