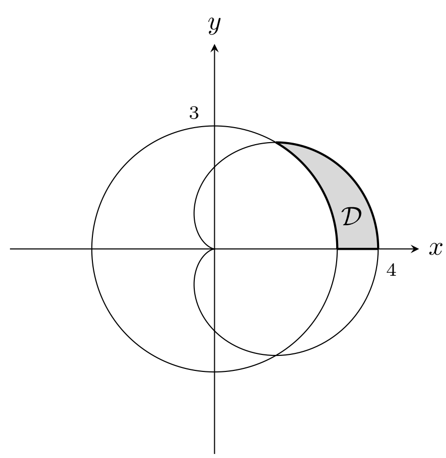
- This exam is worth 100 points and has 6 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/111622 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.
   (a) If vector field $\mathbf{F}(x, y, z)$ possesses continuous second derivatives throughout $\mathbb{R}^3$, then $\nabla \times (\nabla \cdot \mathbf{F}) = \nabla \cdot (\nabla \times \mathbf{F})$.
   (b) The Jacobian, $\dfrac{\partial(x, y)}{\partial(u, v)}$, of the transformation $u = xy, v = xy^2$ is $\dfrac{3}{v}$.
   (c) All gradient vector fields are conservative.
   (d) The curl of $\mathbf{F} = x^2\,\mathbf{i} + (e^z + 2y)\,\mathbf{j} + (xy + yz + xz)\,\mathbf{k}$ lies in the $xy$-plane.
   (e) The vector field $\mathbf{V} = [2x + \cos(yz)]\,\mathbf{i} + x^2y^2\,\mathbf{j} - 2\left(x^2yz + z\right)\mathbf{k}$ is incompressible.

2. [2350/111622 (10 pts)] A thin metal plate (lamina) is placed on the $xy$-plane. The plate is bounded by $x = 1, x = 4$ and $y = 1, y = \sqrt{x}$, and has a density $\rho(x, y) = 3xy$. Carefully follow the directions below.
   (a) (5 pts) Set up, but **do not evaluate**, an appropriate double integral using the order $\mathrm{d}x\,\mathrm{d}y$ that will give the lamina's moment about the $x$-axis.
   (b) (5 pts) Set up, but **do not evaluate**, an appropriate double integral using the order $\mathrm{d}y\,\mathrm{d}x$ that will give the lamina's moment about the $y$-axis.

3. [2350/111622 (20 pts)] You have built a new clubhouse whose floor is in the shape of the shaded region $\mathcal{D}$ in the accompanying figure. The curves shown in the figure are $r = 3$ and $r = 2(1 + \cos\theta)$. The roof of the clubhouse is given by $f(x, y) = \dfrac{8y}{x^2 + y^2}$. Set up and evaluate a double integral to determine the volume of the clubhouse.




4. [2350/111622 (17 pts)] A very thin worm is approximated as the curve $\mathcal{C}$ given by $y = \dfrac{1}{3}x^3$ where $1 \le x \le 2$ ($x$ and $y$ are in cm). The worm's density is given by $\delta(x, y) = \dfrac{9y^2}{x^3}$ g/cm. What is the mass of the worm? Include appropriate units in your answer.

5. [2350/111622 (18 pts)] A friend of yours has built a cool art sculpture in the shape of the surface $\frac{1}{2}x^2 - y - \sqrt{3}z = 0$. To protect it from the elements, a protective coating must be applied to that part of the sculpture lying above the triangular region bounded by the lines $x = \sqrt{5}, y = 0$ and $y = 6\sqrt{3}x$ in the $xy$-plane. The protective coating comes as a liquid in cans, with each can covering 2 square feet. How many cans of the coating must your friend purchase to protect the sculpture? Hint: your answer will be a whole number

6. [2350/111622 (25 pts)] The number of butterflies per cubic foot in a region, $\mathcal{E}$, of space is given by the function $f(x, y, z) = x - z$. The region lies beneath the fourth quadrant $(x > 0, y < 0, z < 0)$ and inside the portion of the sphere $x^2 + y^2 + z^2 = 64$ between the planes $z = -4\sqrt{3}$ and $z = -4$. In parts (b)-(d) set up, **but do not evaluate**, an integral or a sum of integrals that will give the total number of butterflies in the region. For full marks, you must use correct bounds that describe the aforementioned region as stated, not an equivalent one derived from symmetry. Also, be sure to use the standard convention of $0 \le r$, $0 \le \rho$, $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$.

   (a) (4 pts) Make an appropriately labeled sketch in the $rz$-plane (constant $\theta$), clearly indicating the region $\mathcal{E}$.

   (b) (5 pts) Use cylindrical coordinates and the integration order $\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta$.

   (c) (8 pts) Use cylindrical coordinates and the integration order $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$.

   (d) (8 pts) Use spherical coordinates and the integration order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$.