- This exam is worth 100 points and has 5 questions.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"$\times$ 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/101922 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The tangent plane to the surface $z = x^2 + 2xy + 2y^2 - 6x + 8y$ at the point $(10, -7)$ is horizontal.

   (b) There is no real value of k that makes the function $f(x, y) = \begin{cases} \dfrac{2x^2y}{x^3+y^3} & (x, y) \neq (0, 0) \\ k & (x, y) = (0, 0) \end{cases}$ continuous on its domain.

   (c) If $f(x, y) = e^{x^2+3y}$, $x = \sqrt{2}\cos u \sin 2v$, $y = \sqrt{2}\sin 4u \cos v$, then $\dfrac{\partial f}{\partial u} = 10e$ when $u = v = \dfrac{\pi}{4}$.

   (d) The curve in the xy-plane corresponding to all points on the surface $f(x, y) = x^2 - 2x + 4y^2 + 4$ that are 19 units above the xy-plane is a hyperbola.

   (e) The instantaneous rate of change of z with respect to y at the point $(1, 0, 1)$, where $xz^3 + y^2 \ln z + e^x - \cos y + 3xyz = 1$, is 1.

2. [2350/101922 (21 pts)] The centripetal acceleration (m/s^2) of a particle moving in a circle is $a(r, v) = v^2/r$, where v is the velocity (m/s) and r is the radius (m) of the circle.

   (a) (10 pts) Suppose you measure the radius to be roughly 2 m and the velocity to be about 4 m/s with the error in the velocity measurement assumed to be no more than one-half m/s. Use differentials to approximate the maximum error in the measurement of the radius if the error in the acceleration cannot exceed 1 m/s^2.

   (b) (11 pts) Find the second order Taylor polynomial of $a(r, v)$ centered at the point $(2, 4)$. You need not simplify your answer.

3. [2350/101922 (32 pts)] You and a friend are hiking in an area whose elevation is described by $z(x, y) = 1000 + x^3 - 3xy - y^3$. When you arrive at the point P given by $(x, y) = (1, -1)$ your friend begins experiencing altitude sickness.

   (a) (2 pts) What is the elevation at point P?

   (b) (10 pts) What is the slope of the mountain at point P in the direction of $\mathbf{v} = 4\,\mathbf{i} + 3\,\mathbf{j}$?

   (c) (10 pts) Your friend knows that to address the altitude sickness she must must get to a lower elevation as quickly as possible. Give a unit vector in the xy-plane showing the direction in which she should begin walking from point P to make this happen.

   (d) (10 pts) Prior to your friend getting sick, you were walking along the path through P whose projection in the xy-plane is $\mathbf{r}(t) = \cos(\pi t)\,\mathbf{i} + (t-1)^3\,\mathbf{j}$. What was your instantaneous rate of change of elevation with respect to time at P on this path?

4. [2350/101922 (17 pts)] You have thirty dollars to spend on A pounds of almonds and P pounds of peanuts at Nick and Nancy's Nut Nook. Peanuts cost seventy-five cents a pound and almonds are a dollar and a half per pound. The energy, measured in calories, you get from eating the nuts is given by the function $f(A, P) = 3A^{1/2}P^{3/2}$. Hint: Using fractions of dollars rather than decimals will help simplify the algebra.

   (a) (15 pts) Use Lagrange Multipliers to determine how many pounds of each type of nut you should buy in order to maximize the energy you get from eating them.

   (b) (2 pts) What is the maximum value of energy you can get from eating the nuts? You do not have to simplify your answer.

5. [2350/101922 (20 pts)] A gold coin covers the region $x^2 + y^2 \leq 25$ in the xy-plane where the electric charge is given by

$$q(x, y) = x^2 + 2y^2 - x^2y - 3$$

   (a) (15 pts) Are there any relative/local extreme values of charge on the coin? If so, find them. If not, explain why not.

   (b) (5 pts) Without performing any calculations, could the charge on the boundary of the coin be higher and/or lower than any of the values you found in part (a)? Explain briefly.