- This exam is worth 100 points and has 5 questions.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/092122 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.
   (a) A nonzero vector divided by itself is a unit vector.
   (b) If the dot product of two nonzero vectors is zero, then the vectors are scalar multiples of one another.
   (c) If $\mathbf{a}, \mathbf{u}$ and $\mathbf{v}$ are nonzero vectors and $p \neq 0$ is a scalar, then $\mathbf{a} \cdot (p\mathbf{u} \times \mathbf{v}) = \mathbf{a} \times (\mathbf{u} \cdot p\mathbf{v})$
   (d) $\mathbf{a} = \mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are orthogonal.
   (e) If the work done to move a crate 4 meters along a horizontal loading dock by applying a force of 5 newtons is 10 joules, then the force was applied at an angle of $\pi/3$ radians to the dock.

2. [2350/092122 (14 pts)] The surface of the ocean in a wind-free cove is described by the plane $z = 0$. A seabird sitting on the ocean's surface seeks a meal by following the trajectory (path) given by (distances in meters, times in seconds)

$$\mathbf{r}(t) = \left(\frac{2\sqrt{2}}{3}t^{3/2} + 1\right)\mathbf{i} + \left(\frac{2\sqrt{2}}{3}t^{3/2} + 2\right)\mathbf{j} + \left(\frac{1}{2}t^2 - t\right)\mathbf{k}, \quad t \geq 0$$

   (a) (2 pts) Where is the seabird when it dives into the water?
   (b) (2 pts) Where does the seabird exit the water?
   (c) (10 pts) How far does the seabird travel underwater?

3. [2350/092122 (26 pts)] A laser situated on the surface $x^2 - y^2 + z^2 + 2 = 0$ at the point $P(-1, 2, 1)$ is aimed at the point $Q(1, -1, 2)$.
   (a) (4 pts) Name the surface.
   (b) (4 pts) Describe the intersection of the surface with the plane $y = \sqrt{2}$.
   (c) (8 pts) Find a parameterization of the laser beam (assumed to be a straight line). Be sure to include in your answer appropriate values of the parameter that provide the correct orientation of the beam.
   (d) (10 pts) The laser beam hits the surface at another point R.
      i. (5 pts) Find the position vector of R.
      ii. (5 pts) How far is it from the laser to R?

4. [2350/092122 (15 pts)] The unit normal vector to the curve $\mathbf{r}(t) = \left\langle \frac{1}{3}\sin 3t, \sqrt{3}\,t, -\frac{1}{3}\cos 3t \right\rangle$ is $\mathbf{N}(t) = \langle -\sin 3t, 0, \cos 3t \rangle$. Find the equation of the osculating plane when $t = \frac{\pi}{3}$. Write your answer in the form $ax + by + cz = d$.

5. [2350/092122 (35 pts)] A dragonfly is buzzing along a path with a velocity of $\mathbf{v}(t) = \mathbf{i} + 2\,(t-1)\,\mathbf{j} + 0\,\mathbf{k}$, where t is a real number.
   (a) (10 pts) If the dragonfly is at the point $(3, 0, 3)$ when $t = 2$, where is it when $t = 4$?
   (b) (10 pts) Compute the tangential component of the acceleration of the dragonfly's path. Can this be used to determine if there are points on the path where the dragonfly's speed is not changing? If so, find them. If not, explain why not.
   (c) (10 pts) Compute the normal component of the acceleration of the dragonfly's path. Can this be used to determine if there are points on the path where the dragonfly's direction is not changing? If so, find them. If not, explain why not.
   (d) (5 pts) Does the dragonfly's path approach a straight line as $t \to \infty$? Justify your answer.