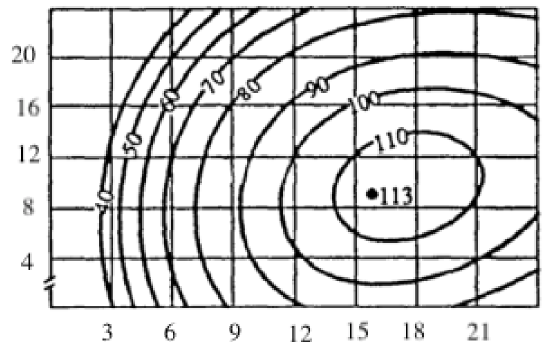
APPM 2350—Final Exam

Saturday, Dec 11th 7:30am-10am 2021

Show all your work and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (TWO sided). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (35 points)
Use the portion of the level curve graph of $g(x, y)$ shown below to answer the following questions. You can assume $g(x, y)$ is a continuous function with continuous partial derivatives.



Do NOT try to find an expression for the actual function $g(x, y)$ (no credit will be given). Use ONLY the information provided on the level curve plot to answer the following questions:

(a) Estimate $g_y(9, 16)$. For credit, *show work* justifying your estimate.

(b) Estimate the derivative of $g(x, y)$ at the point $(9, 16)$ in the direction $-3\mathbf{i} + 4\mathbf{j}$. For credit, *show work* justifying/explaining your estimate.

(c) Estimate $\mathbf{n}$, a unit vector that is normal to the surface $z = g(x, y)$ at the point $(9, 16, 80)$. For credit, *show work* justifying your estimate.

(d) Based on the level curves shown, are there any points in the domain where $\nabla g = \vec{0}$? If not, explain why. If so, give the value of $g(x, y)$ at the point(s).

(e) Let $\mathcal{C}$ be the level curve defined by $g(x, y) = 110$ and oriented counter-clockwise. Determine whether the work done by $\mathbf{G} = \nabla g$ around $\mathcal{C}$ is positive, zero or negative. Justify your answer.

(f) Let $\mathcal{C}$ be the level curve defined by $g(x, y) = 110$. Determine whether the total outward flux of $\mathbf{G} = \nabla g$ through $\mathcal{C}$ is positive, zero or negative. Justify your answer.

(g) **Evaluate** $\int_{\mathcal{C}_1} \nabla g \cdot d\mathbf{r}$, where $\mathcal{C}_1$ is the straight line path from $(x, y) = (21, 12)$ to $(x, y) = (9, 20)$.

Problem 2 (30 points)
The following questions are not related:

(a) Given

$$\mathbf{F} = z\,\mathbf{i} + x\,\mathbf{j} - 3e^{y^2}\arctan(z^2)\,\mathbf{k}$$

Find the outward flux through the surface that consists of the part of the cylinder $x^2 + y^2 = 16$ that lies in the **first octant** between $z = 0$ and $z = 5$. (By outward we mean oriented away from the z-axis. Note that this surface is not closed).

(b) Evaluate

$$\int_{\mathcal{C}} (2x\cos y + 3)\,dx - (x^2\sin y + 2y)\,dy$$

where $\mathcal{C}$ is the path $\mathbf{r}(t) = \cos^3 t\,\mathbf{i} + \sin^3 t\,\mathbf{j}, \quad 0 \le t \le \pi/2$.

CONT'D ON REVERSE SIDE

Problem 3 (25 points)
A metal plate lies in the xy-plane in the region $0 \leq x \leq 4 - y^2$
(i.e the region in quadrants I and IV bounded by the y-axis and the curve $x = 4 - y^2$).

(a) Sketch and shade the region where the plate lies. Label any intercepts.

(b) Find the work done by

$$\mathbf{G} = \left\langle yx^2 - 3y + e^{\cos(x)}, \quad \frac{x^3}{3} - \arctan y \right\rangle$$

counterclockwise once around the boundary of the plate.

(c) Let the temperature at any point on the plate be given by $T(x, y) = y^2 + x^2 - x$. Find the location(s) of the hottest and coldest points on the plate. Show work fully justifying your answer.

Problem 4 (35 points)
The integral

$$\int_0^{2\sqrt{2}} \int_{-\sqrt{8-y^2}}^{\sqrt{8-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{16-x^2-y^2}} dz\, dx\, dy$$

represents the volume of a 3D region, $\mathcal{E}$.

(a) Sketch and shade a 2D cross section of the object in the rz-plane. Label the (r, z) coordinates on all corners of the cross section.

(b) Sketch and shade the projection of the object onto the xy-plane. Label any intercepts.

(c) Set up but DO NOT EVALUATE, equivalent integral(s) to find the volume of the object using:

  (i) Cylindrical coordinates in the order $dz\, dr\, d\theta$

  (ii) Spherical coordinates in the order $d\rho\, d\phi\, d\theta$

(d) Calculate the total outward flux of

$$\boldsymbol{F} = (xz^2 + \tan(y^2 z))\mathbf{i} + (yx^2 - xe^{cos(z)})\mathbf{j} + (zy^2)\mathbf{k}$$

across the entire surface of $\mathcal{E}$.

Problem 5 (25 points)
Let $\mathcal{C}$ be the path parameterized by

$$\mathbf{r}(t) = \langle 2\cos t, \ 2\sin t, \ 3 - \cos t \rangle, \quad 0 \leq t \leq 2\pi$$

.

(a) $\mathcal{C}$ lies in a plane. Find the equation of that plane.

(b) Let
$\mathbf{F}(x, y, z) = \left\langle e^{x^2} + 2z^2, \quad \sinh(y^2), \quad \cos(z^2) + zy \right\rangle$
Find the circulation of $\mathbf{F}(x, y, z)$ around $\mathcal{C}$.

End Of Exam