APPM 2350—Exam 1

Wednesday Sep 22nd, 6:30pm-8pm 2021

Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (18 points)
Consider the three points $P(0,3,1), Q(1,2,-1), R(-1,-2,0)$.

(a) Find the equation of the plane containing the points. Write your answer in standard (i.e. linear) form.
(b) Find the (acute) angle the plane makes with the yz-plane.

Problem 2 (20 points)
Consider the surface $4x - 4y^2 + z^2 = 0$.

(a) Name this surface.
(b) Find the equation of the trace of surface in the plane $z = 2$
(c) Sketch the trace you found in part(b) in 2D. Label any intercepts.
(d) Find the coordinates of the point(s) where the line with the following symmetric equations intersects the surface.

$$\frac{x}{3} = 3 - y = \frac{z-1}{2}$$

Problem 3 (20 points)

A jet travels along the curve of intersection of the surfaces $z = \frac{1}{3}xy$ and $y = \frac{1}{2}x^2$. Assume distances are measured in miles.

(a) Find the total distance the jet travels along this curve starting from the origin and ending at the point $(6, 18, 36)$
(b) Suppose at the point $(2, 2, \frac{4}{3})$ along its path, the jet fires a missile straight ahead at a speed of 12 miles per second. What is the location of the missile 5 seconds after it is fired?

Problem 4 (20 points)
A particle travels along a curve parameterized by

$$\mathbf{r}(t) = \langle 4t, \cos(3t), \sin(3t) \rangle, \quad 0 \le t \le \pi/3$$

where t is time.

(a) At what coordinates (x, y, z), if any, is the particle's unit normal vector, $\mathbf{N}(t)$ parallel to the following plane? Explain/justify your answer.

$$\frac{3}{2}x + y - \sqrt{3}z = \frac{2\pi}{3}$$

(b) At what time(s), if any, is the curvature of the particle's path equal to $\frac{1}{2}$? Explain/justify your answer.

Problem 5 (22 pts)
A car starts at the point $(x, y) = (8, 1)$ when $t = 0$ and moves with **velocity** given by

$$\mathbf{v}(t) = c\langle 1, 2t - 2 \rangle$$

where $t \ge 0$ is the time and $c > 0$ is a constant to be determined.

(a) Find the car's acceleration vector, $\mathbf{a}(t)$.
(b) Find the car's position vector, $\mathbf{r}(t)$
(c) Calculate the normal scalar component of the acceleration, a_N, as a function of t.
(d) Find the time(s) when a_N is largest. Show work justifying your answer.
(e) As an actual industry standard, to avoid rolling over, the normal component of the acceleration for passenger cars must not exceed $1g$, where g is the gravitational acceleration. Find the maximum value of c such that the car won't rollover, as a function of g.