**APPM 2350** **Final Exam** **Fall 2020**

1. [APPM 2350 Exam (30 pts)] Consider the vector field $\mathbf{F} = -\frac{1}{2}y\,\mathbf{i} + \frac{1}{8}x^2y\,\mathbf{j}$. Let $\mathcal{C}$ be the triangle with vertices $(-2, 0)$, $(0, 0)$, $(0, 2)$ oriented counterclockwise.

   (a) (15 pts) By direct calculation, find the circulation of $\mathbf{F}$ along $\mathcal{C}$. Hint: Visualizing the vector field will save some computational effort.

   (b) (15 pts) Using Green's Theorem, calculate the outward flux of $\mathbf{F}$ through $\mathcal{C}$.

2. [APPM 2350 Exam (30 pts)] Consider the force field $\mathbf{F} = 4xe^z\,\mathbf{i} + \cos y\,\mathbf{j} + 2x^2e^z\,\mathbf{k}$.

   (a) (10 pts) Set up, but **do not evaluate**, the integral (in terms of $t$) to find the work done by the force field in moving an object along the curve $\mathcal{C} = \left(\sqrt{t}, \frac{\pi t}{2}, t^2\right)$ for $1 \leq t \leq 3$.

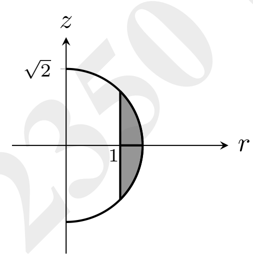
   (b) (15 pts) The integral in part (a) is rather messy but you still need to find the work. Use one of the important Calculus 3 theorems to actually compute the work.

   (c) (5 pts) Now consider the curve $\mathcal{C}$ given by the intersection of the surfaces $x^2 + y^2 = 1$ and $x + y - z = 0$. Find the work done by the force field in moving an object along this curve. Hint: This can be done with **very** little computational effort.

3. [APPM 2350 Exam (30 pts)] The following problems are not related.

   (a) (18 pts) Use Stokes' Theorem to evaluate $\displaystyle\int_{\mathcal{C}} 2z\,\mathrm{d}x + x\,\mathrm{d}y + y^2\,\mathrm{d}z$, where $\mathcal{C}$ is the trace of the surface $z = 4 - x^2 - y^2$ in the $xy$-plane, oriented counterclockwise.

   (b) (12 pts) The charge density in a solid metal ball with radius $\sqrt{2}$ feet is given by $q(\rho, \theta, \phi) = 3\sin\left(\frac{\theta}{2}\right)$ Coulombs per cubic foot. Use spherical coordinates to find the total charge in the portion of the ball whose cross section for an arbitrary $\theta$ is shown in the following figure. Hint: $(\cot x)' = -\csc^2 x$




4. [APPM 2350 Exam (32 pts)] Let $\mathcal{S}$ be the surface of the solid bounded by $z = 4 - y, z = 0, y = 0, x = 0, x = 6$ and let $\mathbf{F} = (x + 1)e^z\mathbf{i} + ye^z\mathbf{j} + e^z\mathbf{k}$. You need to compute the outward flux of $\mathbf{F}$ through $\mathcal{S}$.

   (a) (15 pts) Begin by computing the outward flux of $\mathbf{F}$ through the portion of the surface lying in the $yz$-plane.

   (b) (2 pts) How many more calculations similar to the one in part (a) are required to find the flux? Do not compute it/them, just state how many.

   (c) (15 pts) Rather than evaluating surface integrals to compute the flux, find the outward flux using an appropriate Calculus 3 theorem instead.

5. [APPM 2350 Exam (28 pts)] The elevation of the ground in a certain area is given by $f(x, y) = x^2 - y^2$.

   (a) (2 pts) Identify the quadric surface given by the elevation function.

   (b) (9 pts) You are standing on the surface at the point $P = (5, 10, -75)$ and decide to head in the direction $\mathbf{v} = 3\,\mathbf{i} + 4\mathbf{j}$.

      i. (5 pts) At the instant you begin walking from $P$, determine if your elevation will be increasing or decreasing and find the rate of increase or decrease.

      ii. (4 pts) From the point $P$, what is the largest (in absolute value) rate of change in elevation that you can attain?

   (c) (5 pts) Use the chain rule to find the rate of change of elevation along the path $(x, y) = \left(t, \frac{1}{2}t^2 - 1\right)$ when $t = 2$.

   (d) (12 pts) Suppose you decide to hike along a path whose $(x, y)$ coordinates are constrained to satisfy $2y - x^2 = -2$. Find the value and location(s) of highest elevation you will reach.