1. [APPM 2350 Exam (19 pts)] Sketch the region of integration and evaluate the following integral by switching to polar coordinates.

$$\int_1^2 \int_{-\sqrt{2y-y^2}}^0 \frac{y}{x^2+y^2}\,\mathrm{d}x\,\mathrm{d}y$$

   Potentially helpful information: $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\quad \sin^2 x = \frac{1}{2}(1-\cos 2x)$

2. [APPM 2350 Exam (20 pts)] The density of pollen particles is given by $\rho(x,y) = 72(x+y)e^{x^2-y^2}$ g/cm^2. By making an appropriate change of variables, determine the mass of pollen contained in the rectangle, $\mathcal{R}$, enclosed by the lines $x-y=0, x-y=2, x+y=3, x+y=6$.

3. [APPM 2350 Exam (21 pts)] Consider the region $\mathcal{W}$ below the fourth quadrant and inside the sphere $x^2+y^2+z^2=36$ between the planes $z=-3$ and $z=-3\sqrt{3}$. We want to find $B = \iiint_{\mathcal{W}} xyz\,\mathrm{d}V$.

   (a) Set up, but **DO NOT EVALUATE** the integral(s) necessary to compute B in rectangular/Cartesian coordinates using the order $\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$.

   (b) Set up, but **DO NOT EVALUATE** the integral(s) necessary to compute B in cylindrical coordinates using the order $\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta$.

   (c) Set up, but **DO NOT EVALUATE** the integral(s) necessary to compute B in spherical coordinates using the order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$.

4. [APPM 2350 Exam (20 pts)] Evaluate $\iint_{\mathcal{S}} 48\sqrt{3}\,yz\,\mathrm{d}S$ where $\mathcal{S}$ is the portion of the surface

$$\sqrt{3}x = y + 2z^2 \ \text{ with } -\sqrt{3}/2 \le y \le 0, -y \le z \le \sqrt{3}/2$$

5. [APPM 2350 Exam (20 pts)] The electric charge q at a point (x,y,z) in space is equal to the square of the distance from the point to the origin. Find the average value of the charge on a wire that lies along the curve $\mathcal{C} = \left(\sin\left(\pi t^2\right), \sqrt{3}\pi t^2, \cos\left(\pi t^2\right)\right)$, $t>0$ between the points $\left(0, \sqrt{3}\pi, -1\right)$ and $\left(0, 4\sqrt{3}\pi, 1\right)$.