1. [APPM 2350 Exam (22 pts)] The following problems are not related.

   (a) (10 pts) The critical points of the function $f(x, y) = x^3 - 12xy + 8y^3$ are $(0, 0)$ and $(2, 1)$. Classify each of these as either a saddle point or local extremum. Justify your answer mathematically.

   (b) (12 pts) Kalkthree Regional Park consists of the boundary and interior of the triangle described by $x = 0$, $y = 0$, and $x + y = 4$. If the elevation of the park is given by $f(x, y) = x^2 + 2xy - y^2 - 4x$, find the highest and lowest points in the park as well as their locations.

2. [APPM 2350 Exam (20 pts)] You are contemplating building a house in the shape of a box (without a roof for now) to provide a quiet spot to do Calculus 3 homework. The house is to have a volume of 81 m^3. The wood for the bottom of the house costs 6 times as much (per unit area) as the wood for the sides. Using Lagrange Multipliers, find the dimensions of the house that will minimize the cost of the lumber.

3. [APPM 2350 Exam (34 pts)] The following problems are not related.

   (a) (8 pts) The ideal gas law for a particular gas is given by $T = PV$, where T is temperature, P is pressure, and V is volume. You have an indestructible balloon of volume 1 m^3 at a temperature of 300 Kelvin filled with this gas. You want to test the limits of the indestructible balloon so you place it in an oven. The temperature of the gas is immediately increasing at a rate of 5 Kelvin per second. The volume is increasing at a rate of 0.01 cubic meters per second. At this instant, how fast is the pressure changing?

   (b) (8 pts) If $z = x + y + \sqrt{xy}$ and $x = te^s$, $y = s^2 + t^2$, find z_t when $s = 0$ and $t = 4$.

   (c) (18 pts) Let $f(x, y) = \ln(2x + y)$.

      i. (4 pts) Find and sketch the domain of $f(x, y)$.

      ii. (4 pts) Find and sketch the level curve corresponding to $f(x, y) = 2$. Label all, if any, intercepts.

      iii. (2 pts) Evaluate $\lim\limits_{(x,y)\to(e,e)} f(x, y)$.

      iv. (8 pts) Find the second order Taylor polynomial for $f(x, y)$ centered at $(1, 2)$. Simplify your answer.

4. [APPM 2350 Exam (24 pts)] The following problems are not related.

   (a) (5 pts) Given the surface $3x^2 + y^2 - z^2 = -5$, find the equation of the tangent plane at the point $(1, 1, -3)$. Write your simplified answer in the form $ax + by + cz = d$.

   (b) (19 pts) The depth of a lake is given by the equation $h(x, y) = 2x^4 + 3y^2 - 10$. You and some friends are in a boat on the surface of the lake, which is located in the xy–plane. All length/depth units are in meters.

      i. (8 pts) At the point $(1, -1, 0)$ you are heading in the direction $2\mathbf{i} - \mathbf{j}$. Is the lake getting shallower or deeper? At what rate?

      ii. (8 pts) From the point $(1, -1, 0)$, in what direction(s) should you set sail so as to follow the level curve through the point? Write your answer as a unit vector(s).

      iii. (3 pts) Your friends are really in a hurry to get to deeper water. They ask you to set sail from the point $(1, -1, 0)$ in a direction that will make the depth of the lake change at a rate of 12 m/m. What should you tell them? Justify your answer mathematically.