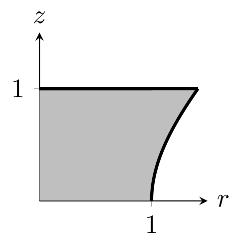
Write on the front of your bluebook a grading key (using the printed lines), your name, your lecture section number and instructor. This exam is worth 100 points and has 5 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You are taking this exam in a proctored and honor code enforced environment. Thus, no notes/papers, calculators, cell phones, or other electronic devices are permitted.

NOTE: Any integrals that need to be evaluated will require integration techniques no more complicated than *u*-substitution or the use of trigonometric identities.

1. [20 pts] Use a change of variables to find the volume of the region under the graph of $f(x, y) = 4xy$ lying over the portion of the first quadrant bounded by the parabolas $x = y^2 - 5$, $x = y^2 - 3$, $x = -y^2 + 6$, and $x = -y^2 + 10$.

2. [20 pts] The density of soot particles inside a smokestack is given by $\delta(x, y, z) = 12x^2z$ particles per cubic meter. The smokestack is in the shape of the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$ for $0 \le z \le 1$. A cross section of half of the smokestack is shown in the accompanying $rz-$plane (constant θ plane). Find the total number (which may not be an integer) of soot particles in the smokestack.



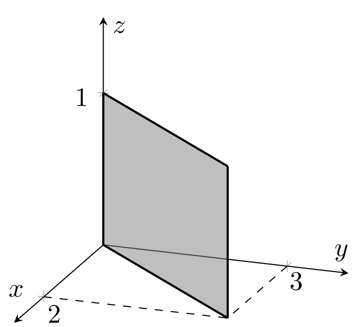
3. [20 pts] A spherical tank of radius 4 feet is centered at the origin and is partially filled with oil. A straight measuring stick enters the sphere at the point $(\rho, \theta, \phi) = (4, 0, 0)$ and measures the depth of the oil as 2 feet.

   (a) Make a sketch of the situation in the $xz-$plane, being sure to show the location of the oil.

   (b) Set up, but **do not** evaluate the integral to find the volume of oil in the tank using the integration order $\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$.

   (c) Set up, but **do not** evaluate the integral to find the volume of oil in the tank using the integration order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$.

4. [20 pts] A thin metal rectangular plate is standing vertically in the first octant as shown. If the density of the metal is $1 + 6x + 8y(z + 1)$ pounds per square foot, how much does the plate weigh?



5. [20 pts] You have a cool new "exponential" curtain covering a window in your apartment. It touches the floor ($xy-$plane) along the curve $y = e^{x/2}$ lying between $x = \ln 32$ and $x = \ln 60$. The top of the curtain reaches up to the surface $z = 3e^x$. All units are in feet. The fabric making up the curtain is sensitive to sunlight so you need to purchase some UV protection to spray on the side of the curtain facing the window. The UV protector comes in spray cans that each cover 100 square feet. Use a line integral to determine how many full cans of the spray you must purchase to protect your curtain.

FORMULAS ON BACK

CHANGE OF VARIABLES/SUBSTITUTIONS IN MULTIPLE INTEGRALS

$$\iint_{\mathcal{R}} f(x,y)\,\mathrm{d}A = \iint_{\mathcal{S}} f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| \mathrm{d}u\,\mathrm{d}v \quad \text{where} \quad J(u,v) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

POLAR COORDINATES $\quad x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad \mathrm{d}A = \mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$

Coordinate Conversions

Cylindrical to Rectangular	Spherical to Rectangular	Spherical to Cylindrical
$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$	$r = \rho\sin\phi$
$y = r\sin\theta$	$y = \rho\sin\phi\sin\theta$	$z = \rho\cos\phi$
$z = z$	$z = \rho\cos\phi$	$\theta = \theta$

$$\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$$

MASS, MOMENTS, AND CENTER OF MASS $\quad$ Mass $M = \iint_R \delta\,\mathrm{d}A \quad$ Moments $M_x = \iint_R y\,\delta\,\mathrm{d}A \quad M_y = \iint_R x\,\delta\,\mathrm{d}A \quad$ Center of mass $\overline{x} = \frac{M_y}{M} \quad \overline{y} = \frac{M_x}{M}$

SURFACE AREA OF LEVEL SURFACE $g(x,y,z) = c \quad S = \iint_{\mathcal{S}} \mathrm{d}S = \iint_{\mathcal{R}} \frac{\|\nabla g\|}{|\nabla g \cdot \mathbf{p}|}\,\mathrm{d}A$

FUN TRIGONOMETRY FACTS $\quad \sin^2 x = \frac{1-\cos 2x}{2} \quad \cos^2 x = \frac{1+\cos 2x}{2}$