Write on the front of your bluebook a grading key (using the printed lines), your name, your lecture section number and instructor. This exam is worth 100 points and has 5 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You are taking this exam in a proctored and honor code enforced environment. Thus, no notes/papers, calculators, cell phones, or other electronic devices are permitted.

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1. [27 pts] You and a group of friends are hiking in Calcthree Municipal Park. The park's boundaries are given by $|x| = 2$ and $|y| = 2$ and the elevation above sea level in the park is described by the function $z(x, y) = 10 + x^3 - 3x + 2y^2 - y^4$.

   (a) [3 pts] Is there a highest point in the park to which you and your friends can hike? Don't try to find it, just decide if there is one and in a single sentence justify your answer.

   (b) [24 pts] Provide mathematical justifications for your answers to the following questions:

   i. [6 pts] You are standing at the point $(1, -1, 9)$. Are you in a valley, on a hilltop, in a saddle or none of these?

   ii. [6 pts] You spot a bear whose (x, y) coordinates are $(-1, 1)$. Is it above or below you? Is it on a hilltop?

   iii. [6 pts] Your friends are standing at the location in the park that is directly above the origin. They also spot the bear. To avoid it they decide to head toward your location. When they start out, will they be ascending or descending? At what rate?

   iv. [6 pts] If your friends want to lose elevation at the fastest rate, in which direction should they begin walking from the origin to accomplish this? Write your answer as a unit vector.

2. [28 pts] Let $g(x, y) = \sqrt{4 - 3x + 2y}$.

   (a) [6 pts] Find and sketch the domain of $g(x, y)$. Label any intercepts.

   (b) [2 pts] Find the range of $g(x, y)$, writing your answer using interval notation.

   (c) [3 pts] Where is $g(x, y)$ continuous?

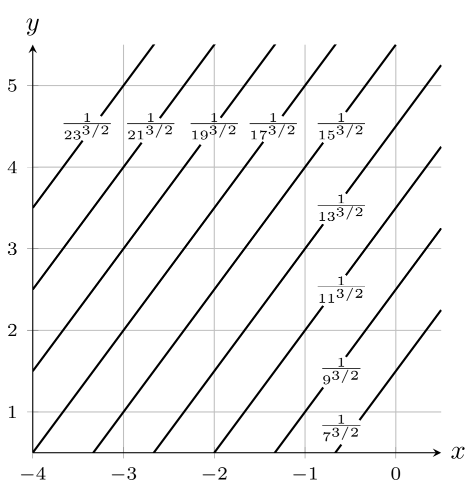
   (d) [3 pts] Find $\lim\limits_{(x,y)\to(0,0)} g(x, y)$ or show that is does not exist.

   (e) [8 pts] Find the first order Taylor polynomial (linearization) for $g(x, y)$ centered at $(-2, 3)$. Write your final answer in the form $T_1(x, y) = ax + by + c$.

   (f) [6 pts] The following figure shows a contour map of the function $h(x, y) = (4 - 3x + 2y)^{-3/2}$. Use this and the fact that

$$g_{xx}(x, y) = \frac{-9}{4(4 - 3x + 2y)^{3/2}} \qquad g_{xy}(x, y) = \frac{-3}{2(4 - 3x + 2y)^{3/2}} \qquad g_{yy}(x, y) = \frac{-1}{(4 - 3x + 2y)^{3/2}}$$

   to find an upper bound on the absolute value of the error in the linearization of $g(x, y)$ if $|x + 2| < 1$ and $|y - 3| < 2$.



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3. [20 pts] The temperature in a region of space is given by $T(x, y, z) = 1000 + x^2 + y^2 + z^2$. The super-duper Rate-O-Change meter on board your spaceship gives a readout of the instantaneous rate of change of temperature with respect to any variable you enter into it. To receive credit, you must use Calculus 3 concepts to answer this question. Improper notation will be penalized.

   (a) [10 pts] Suppose you know the following information about the path of your spaceship:

   $$\mathbf{r}(0) = \mathbf{k} \qquad \mathbf{r}(1) = 2\,\mathbf{i} + \mathbf{k} \qquad \mathbf{r}'(0) = -4\,\mathbf{i} + 2\,\mathbf{j} + 7\,\mathbf{k} \qquad \mathbf{r}'(1) = 2\,\mathbf{i} + 2\pi\,\mathbf{j}$$

   What does your meter read when you enter t into it and you are at the point $(2, 0, 1)$?

   (b) [10 pts] Now suppose your spaceship's position is given by $x(v, w) = (v^2 + w^2)$, $y(u, v) = \ln(uv)$, $z(u, v) = e^{2u+4v}$. What does your meter read if you enter u into it when $u = 2, v = \frac{1}{2}$ and $w = 1$?

4. [10 pts] The body mass index (B) as a function of weight (W, kg) and height (H, m) is given by $B = W/H^2$. Suppose that for a 2 m tall, 100 kg person you know that the height measurement is 0.01 m too high. If you want the body mass index to have an error no greater than 0.25, use differentials to determine the maximum error that can be present in the weight measurement.

5. [15 pts] The satisfaction you get from eating x slices of bacon, y eggs and z potatoes is given by the satisfaction function $s(x, y, z) = \sqrt{xyz}$. Suppose you have six dollars to spend and a rather hearty appetite. If each slice of bacon costs 50 cents, each egg costs 25 cents and each potato costs 10 cents, determine how many bacon slices, eggs and potatoes you should purchase to be the most satisfied. You must use Calculus III concepts to obtain any credit on this problem.

PROJECTION; DISTANCE FROM POINT S TO LINE PARALLEL TO $\mathbf{v}$ CONTAINING POINT P; DISTANCE FROM POINT S TO PLANE WITH NORMAL $\mathbf{n}$ CONTAINING POINT P

$$\operatorname{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a} \qquad d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{\|\mathbf{n}\|}\right|$$

ARC LENGTH, FRENET FORMULAS, AND TANGENTIAL AND NORMAL ACCELERATION COMPONENTS

$$\mathrm{d}s = \|\mathbf{v}\|\,\mathrm{d}t \qquad \mathbf{T} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \qquad \mathbf{N} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}s}{\|\mathrm{d}\mathbf{T}/\mathrm{d}s\|} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}t}{\|\mathrm{d}\mathbf{T}/\mathrm{d}t\|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s} = \kappa\mathbf{N} \qquad \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s} = -\tau\mathbf{N} \qquad \kappa = \left\|\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s}\right\| = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|^3} = \frac{|f''(x)|}{\left\{1+[f'(x)]^2\right\}^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \qquad \tau = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad a_T = \frac{\mathrm{d}\|\mathbf{v}\|}{\mathrm{d}t} \qquad a_N = \kappa\|\mathbf{v}\|^2 = \sqrt{\|\mathbf{a}\|^2 - a_T^2}$$

DIRECTIONAL DERIVATIVE, LAGRANGE MULTIPLIERS $\quad D_{\mathbf{u}}f = \dfrac{\mathrm{d}f}{\mathrm{d}s} = \nabla f\cdot\mathbf{u} \qquad \nabla f = \lambda\nabla g, \quad g = 0$

SECOND DERIVATIVES TEST: Suppose $f(x, y)$ and its first and second partial derivatives are continuous in a disk centered at (a, b) and $f_x(a, b) = f_y(a, b) = 0$. Let $D = f_{xx}f_{yy} - f_{xy}^2$.

- If $D > 0$ and $f_{xx} < 0$ at (a, b), then f has a local maximum at (a, b).
- If $D > 0$ and $f_{xx} > 0$ at (a, b), then f has a local minimum at (a, b).
- If $D < 0$ at (a, b), then f has a saddle point at (a, b).
- If $D = 0$ at (a, b), then the test is inconclusive.

TAYLOR'S FORMULA [at the point (x_0, y_0)]

$$f(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) + \frac{1}{2!}\left[f_{xx}(x_0,y_0)(x-x_0)^2 + 2f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f_{yy}(x_0,y_0)(y-y_0)^2\right]$$
$$+\frac{1}{3!}\left[f_{xxx}(x_0,y_0)(x-x_0)^3 + 3f_{xxy}(x_0,y_0)(x-x_0)^2(y-y_0) + 3f_{xyy}(x_0,y_0)(x-x_0)(y-y_0)^2 + f_{yyy}(x_0,y_0)(y-y_0)^3\right] + \cdots$$

ERROR IN LINEAR APPROXIMATION $\quad |E(x,y)| \le \dfrac{1}{2!}M\left(|x-x_0| + |y-y_0|\right)^2,\ $ where $\max\{|f_{xx}|, |f_{xy}|, |f_{yy}|\} \le M$

ERROR IN QUADRATIC APPROXIMATION $\quad |E(x,y)| \le \dfrac{1}{3!}M\left(|x-x_0| + |y-y_0|\right)^3,\ $ where $\max\{|f_{xxx}|, |f_{xxy}|, |f_{xyy}|, |f_{yyy}|\} \le M$