Write on the front of your bluebook a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 4 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You are taking this exam in a proctored and honor code enforced environment. Thus, no notes/papers, calculators, cell phones, or other electronic devices are permitted.

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1. [12 pts] In your bluebook, write the word **TRUE** if the statement is true or write the word **FALSE** if the statement is false. No justification needed and no partial credit given.

   (a) If the velocity and acceleration vectors of a curved path are both nonzero, but scalar multiples of one another at a point, then the normal component of the acceleration is zero at that point.

   (b) $-x^2 + y^2 - 2z^2 + 2x + 2y + 4z - 1 = 0$ represents an elliptic cone with vertex at $(-1, 1, -1)$.

   (c) For any two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, $\text{comp}_{\mathbf{a}}\mathbf{b} = (\text{proj}_{\mathbf{a}}\mathbf{b})\,\mathbf{a}$

   (d) A particle moving along the path $\mathbf{r}(t) = (-4 - \sin 2t)\,\mathbf{i} - \mathbf{j} + (5 + \cos 2t)\,\mathbf{k}$ experiences constant curvature.

2. [30 pts] A mosquito is buzzing along the path $\mathbf{r}(t) = t\,\mathbf{i} + \frac{2\sqrt{2}}{3}t^{3/2}\,\mathbf{j} + \frac{1}{2}t^2\,\mathbf{k},\ t \geq 0$ with distance measured in yards and time in seconds.

   (a) [10 pts] Starting from time $t = 0$, how long does it take the mosquito to travel 12 yards along this path?

   (b) [10 pts] After 1 second, how far is the mosquito from its starting position at $t = 0$?

   (c) [10 pts] Now suppose that after flying along the aforementioned path for 2 seconds, the mosquito notices a bat approaching from behind. In an effort to avoid becoming the bat's dinner, from this point the mosquito leaves the original path, flying straight ahead along a line at a constant speed of 1 yard/sec. Find the mosquito's coordinates after it travels along this line for 3 seconds.

3. [28 pts] Consider the path given by $\mathbf{r}(t) = \sqrt{3}\,t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k},\ \ t \geq 0.$

   (a) [14 pts] Find the value(s) of t in the interval $[0, 2\pi]$ where the osculating plane is parallel to the plane $-\frac{1}{2}x + \frac{3}{4}y - \frac{\sqrt{3}}{4}z = 1.$

   (b) [14 pts] Find the equation of the normal plane to the path when $t = \pi/6$. Write your answer in the form $ax + by + cz = d$.

4. [30 pts] You are playing indoors with a paper airplane, the velocity of which is given by $\mathbf{v}(t) = (1 - 3\cos t)\,\mathbf{i} + 3\sin t\,\mathbf{k}$.

   (a) [10 pts] Find the tangential component of the acceleration vector.

   (b) [10 pts] If the initial position of the airplane is $\mathbf{r}(0) = \mathbf{i} - \mathbf{j} + \mathbf{k}$, find an expression for the path of the airplane.

   (c) [10 pts] Look out! On its maiden voyage the airplane crashes into the ceiling, considered as a plane parallel to the $xy-$plane. The crash occurs when $t = \pi/2$. Find the acute angle at which the airplane hits the ceiling.

PROJECTIONS, DISTANCES FROM POINT S TO LINE CONTAINING POINT P, AND S TO PLANE WITH NORMAL $\mathbf{n}$

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a} \qquad d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{\|\mathbf{n}\|}\right|$$

ARC LENGTH, FRENET FORMULAS, AND TANGENTIAL AND NORMAL ACCELERATION COMPONENTS

$$\mathrm{d}s = \|\mathbf{v}\|\,\mathrm{d}t \qquad \mathbf{T} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \qquad \mathbf{N} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}s}{\|\mathrm{d}\mathbf{T}/\mathrm{d}s\|} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}t}{\|\mathrm{d}\mathbf{T}/\mathrm{d}t\|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s} = \kappa\mathbf{N} \quad \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s} = -\tau\mathbf{N} \quad \kappa = \left\|\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s}\right\| = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|^3} = \frac{|f''(x)|}{\left\{1 + [f'(x)]^2\right\}^{3/2}} = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \quad \tau = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad a_T = \frac{\mathrm{d}\|\mathbf{v}\|}{\mathrm{d}t} \qquad a_N = \kappa\|\mathbf{v}\|^2 = \sqrt{\|\mathbf{a}\|^2 - a_T^2}$$