APPM 2350—Section Exam 3—140 points

Wednesday November 28, 6:00pm – 7:30pm, 2018

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section/time (4) your instructor's name, and (5) a grading table. Text books, class notes, and calculators are NOT permitted. A one-page one-sided crib sheet is allowed.

Problem 1 – True/False: (20 points)
For the following true/false questions, write TRUE (for always true) or FALSE (if not always true). Your work will not be graded.

(a) The force field $\mathbf{F}(x, y) = \langle -16 - 5y - 3x^2, -7 - 5x - 4y^2 \rangle$ acts mostly against the movement of a particle that travels once counterclockwise around the triangle with corners $(1, 0)$, $(0, 1)$, $(-1, 0)$ because the work done is negative.

(b) For any function $f(x, y)$ continuous on all of $\mathbb{R}^2$, $\displaystyle\int_0^1 \int_0^x f(x, y)\, dy\, dx = \int_0^1 \int_0^y f(x, y)\, dx\, dy$.

(c) A line parallel to the y-axis through the center of mass of a closed, connected, two-dimensional lamina in the x-y plane divides the lamina into subregions of equal area.

(d) Suppose the curve C is the straight line path from the origin to the point (π, π), then the line integral $\int_C f(x, y)\, \mathrm{d}s$ can be written $\int_0^\pi f(t, t)\, \mathrm{d}t$.

Problem 2 – Short Answer Questions: (40 points)
For the questions in this problem, show all your work and clearly $\boxed{\text{box}}$ your final answer. Partial credit may be given.

(a) The integral

$$I = \int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} dz\, dx\, dy$$

calculates the volume of a 3D object in Cartesian coordinates. However, it's difficult to evaluate in these coordinates. Convert the integral I to an equivalent integral (or integrals) in spherical coordinates. (You DO NOT need to evaluate).

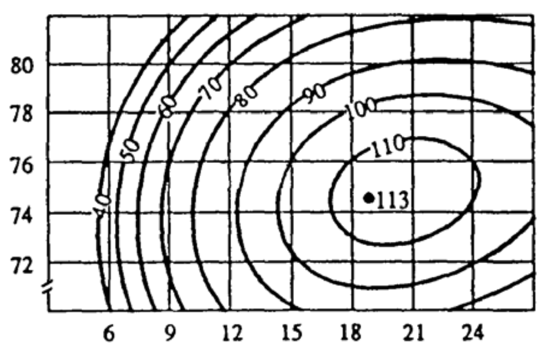

FIGURE 1. **Problems 2b, 2c, and 2d utilize this level curve (contour) graph of the function $g(x, y)$. In all these problems, the vector field $\mathbf{G}(x, y) = \nabla g(x, y)$ is used.**

(b) Let C be the level curve defined by $g(x, y) = 110$ and oriented counter-clockwise. Determine whether the work done by $\mathbf{G}$ around C is positive, zero, or negative. Justify your answer.

(c) Let C be the level curve defined by $g(x, y) = 110$ and oriented counter-clockwise. Determine whether the total flux of $\mathbf{G}$ out of C is positive, zero, or negative. Justify your answer.

(d) Let C' be the straight line path $\mathbf{r}(t)$ from $(x, y) = (18, 76)$ to $(x, y) = (9, 80)$. Evaluate the work integral $\int_{C'} \mathbf{G} \cdot \mathrm{d}\mathbf{r}$.

Problem 3: (40 points)
On a certain day, the density of lightning strikes in the Colorado Rocky Mountains was found to be well-approximated by the function

$$\delta(x,y) = \left(x^2+y^2\right)^{3/2}\left[1+12\tan^{-1}(y/x)\right]$$

strikes per unit area. Some colleagues of yours need to calculate the total number of lightning strikes N that day in a certain area to help determine the risk of new forest fires. They have decided that this can be accomplished by computing the value of

$$N = \int_{\sqrt{3}/2}^{1}\int_{\sqrt{1-x^2}}^{x/\sqrt{3}} \delta(x,y)\,\mathrm{d}y\,\mathrm{d}x + \int_{1}^{\sqrt{3}}\int_{0}^{x/\sqrt{3}} \delta(x,y)\,\mathrm{d}y\,\mathrm{d}x + \int_{\sqrt{3}}^{2}\int_{0}^{\sqrt{4-x^2}} \delta(x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

However, since they took Calculus 3 many moons ago, they cannot recall how to do this calculation. Come to their assistance by determining the total number N of lightning strikes in their area of interest, simplifying your final answer. *Hint: draw the region of integration. Another coordinate system will be useful.*

Problem 4: (40 points)
Your mechanical engineering friend designs a new part using CAD software. An important component of the part is determined by the curve C that is given by the intersection of the surfaces $y = x^2 + 9$ and $z = 3$ in three dimensions.

(a) A wire is placed along the curve C from $(2,13,3)$ to $(3,18,3)$ whose density at a point on the wire is given by its x-coordinate. Unfortunately, the CAD software is not capable of determining the mass of the wire so, your friend needs your help. Find the mass of the wire.

(b) Let $\mathbf{G}(x,y,z) = 2xy\mathbf{i} + (x^2+2z)\mathbf{j} + 2y\mathbf{k}$ be a vector force field. Show that $\mathbf{G}$ is conservative and find its potential function $g(x,y,z)$ so that $\mathbf{G}(x,y,z) = \nabla g(x,y,z)$.

(c) Find the work done on the particle by the force field $\mathbf{G}$ when moving along C from $(0,9,3)$ to $(2,13,3)$.