ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section/time (4) your instructor’s name, and (5) a grading table. Text books, class notes, and calculators are NOT permitted. A one-page one-sided crib sheet is allowed.

Problem 1 – True/False: (20 points)
For the following true/false questions, write TRUE (for always true) or FALSE (if not always true). Your work will not be graded.

(a) The force field \( \mathbf{F}(x, y) = \langle -16 - 5y - 3x^2, -7 - 5x - 4y^2 \rangle \) acts mostly against the movement of a particle that travels once counterclockwise around the triangle with corners \((1, 0), (0, 1), (-1, 0)\) because the work done is negative.

(b) For any function \( f(x, y) \) continuous on all of \( \mathbb{R}^2 \), \( \int_0^1 \int_0^x f(x, y) \, dy \, dx = \int_0^1 \int_0^y f(x, y) \, dx \, dy \).

(c) A line parallel to the \( y \)-axis through the center of mass of a closed, connected, two-dimensional lamina in the \( x-y \) plane divides the lamina into subregions of equal area.

(d) Suppose the curve \( C \) is the straight line path from the origin to the point \((\pi, \pi)\), then the line integral \( \int_C \mathbf{G} \cdot d\mathbf{r} \) can be written \( \int_0^\pi f(t, t) \, dt \).

Problem 2 – Short Answer Questions: (40 points)
For the questions in this problem, show all your work and clearly box your final answer. Partial credit may be given.

(a) The integral
\[
I = \int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} dz \, dx \, dy
\]
calculates the volume of a 3D object in Cartesian coordinates. However, it’s difficult to evaluate in these coordinates. Convert the integral \( I \) to an equivalent integral (or integrals) in spherical coordinates. (You DO NOT need to evaluate).

(b) Let \( C \) be the level curve defined by \( g(x, y) = 110 \) and oriented counter-clockwise. Determine whether the work done by \( \mathbf{G} \) around \( C \) is positive, zero, or negative. Justify your answer.

(c) Let \( C \) be the level curve defined by \( g(x, y) = 110 \) and oriented counter-clockwise. Determine whether the total flux of \( \mathbf{G} \) out of \( C \) is positive, zero, or negative. Justify your answer.

(d) Let \( C' \) be the straight line path \( \mathbf{r}(t) \) from \((x, y) = (18, 76)\) to \((x, y) = (9, 80)\). Evaluate the work integral \( \int_{C'} \mathbf{G} \cdot d\mathbf{r} \).
Problem 3: (40 points)
On a certain day, the density of lightning strikes in the Colorado Rocky Mountains was found to be well-approximated by the function

\[ \delta(x, y) = (x^2 + y^2)^{3/2} \left[ 1 + 12 \tan^{-1} \left( \frac{y}{x} \right) \right] \]

strikes per unit area. Some colleagues of yours need to calculate the total number of lightning strikes \( N \) that day in a certain area to help determine the risk of new forest fires. They have decided that this can be accomplished by computing the value of

\[ N = \int_{\sqrt{3}/2}^{1} \int_{x/\sqrt{3}}^{\sqrt{3}/2} \delta(x, y) \, dy \, dx + \int_{1}^{\sqrt{3}} \int_{0}^{x/\sqrt{3}} \delta(x, y) \, dy \, dx + \int_{\sqrt{3}}^{2} \int_{0}^{\sqrt{4-x^2}} \delta(x, y) \, dy \, dx. \]

However, since they took Calculus 3 many moons ago, they cannot recall how to do this calculation. Come to their assistance by determining the total number \( N \) of lightning strikes in their area of interest, simplifying your final answer. \( \text{Hint: draw the region of integration. Another coordinate system will be useful.} \)

Problem 4: (40 points)
Your mechanical engineering friend designs a new part using CAD software. An important component of the part is determined by the curve \( C \) that is given by the intersection of the surfaces \( y = x^2 + 9 \) and \( z = 3 \) in three dimensions.

(a) A wire is placed along the curve \( C \) from \((2, 13, 3)\) to \((3, 18, 3)\) whose density at a point on the wire is given by its \( x \)-coordinate. Unfortunately, the CAD software is not capable of determining the mass of the wire so, your friend needs your help. Find the mass of the wire.

(b) Let \( \mathbf{G}(x, y, z) = 2xy \mathbf{i} + (x^2 + 2z) \mathbf{j} + 2y \mathbf{k} \) be a vector force field. Show that \( \mathbf{G} \) is conservative and find its potential function \( g(x, y, z) \) so that \( \mathbf{G}(x, y, z) = \nabla g(x, y, z) \).

(c) Find the work done on the particle by the force field \( \mathbf{G} \) when moving along \( C \) from \((0, 9, 3)\) to \((2, 13, 3)\).