APPM 2350—Section Exam 2—140 points

Wednesday October 24, 6:00pm – 7:30pm, 2018

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section/time (4) your instructor's name, and (5) a grading table. Text books, class notes, and calculators are NOT permitted. A one-page one-sided crib sheet is allowed.

Problem 1 – True/False: (20 points)
For the following true/false questions, write TRUE (for always true) or FALSE (if not always true). Your work will not be graded.

(a) If $f(x,y) \to L$ as $(x,y) \to (a,b)$ along *every straight line* passing through (a,b), then
$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

(b) Given $f(x(r,s), y(r), z(r,s,t))$ then
$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{dy}{dr} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$$

(c) Given $f(x,y,z) = xyz$ the smallest (ie most negative) derivative of $f(x,y,z)$ at the point $(1,2,3)$ (out of all possible directions) is -7.

(d) The vector $\nabla f(a,b)$ is normal to the graph of $z = f(x,y)$ at the point $(a,b,f(a,b))$.

Problem 2 – Short Answer Questions: (60 points)
For the questions in this problem, show all your work and clearly $\boxed{\text{box}}$ your final answer. Partial credit may be given.

(a) Find $\partial z/\partial y$ for $z = z(x,y)$ that satisfies the relation $\ln(xz) + \sin(yz) - \frac{z^2}{2} = \pi$.

(b) Find a value of the constant c that makes the following function continuous at $(0,0)$:
$$f(x,y) = \begin{cases} \dfrac{e^{x^2+y^2} - 1}{3(x^2+y^2)} & \text{if } (x,y) \neq (0,0) \\ c & \text{if } (x,y) = (0,0) \end{cases}$$

(c) If $g(s,t) = f(x(s,t), y(s,t))$ where $x(s,t) = s + t$ and $y(s,t) = s - t$, find a simplified expression for g_{tt} in terms of derivatives of f. Assume all partial derivatives involved exist and are continuous.

(d) At a certain point P on a mountain, a skier looks directly east and measures a 30° drop-off (decline), then looks directly north and measures a $45°$ rise (incline). Using these two measurements of the mountain topography, find a vector that points in the direction of steepest **descent** from the point P.

SEE PROBLEMS ON THE OTHER SIDE

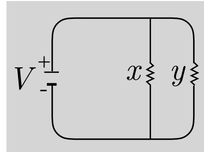

FIGURE 1. Two-resistor circuit with resistors x and y in parallel.

Problem 3: (30 points)
The total resistance R of two resistors that are connected in parallel, each with resistances x and y such as in the circuit in Fig. 1, are related according to

$$\frac{1}{R} = \frac{1}{x} + \frac{1}{y}.$$

(a) If x, y, and R are allowed to vary, find an expression for the differential dR in terms of the differentials dx and dy.
(b) You have designed a two-resistor circuit like the one in Fig. 1 to have resistances $x = 10$ Ohms and $y = 40$ Ohms, but there is always some variation in manufacturing and the resistors received by your firm will probably not have these exact values. Will the total resistance R be more sensitive to variation in x or to variation in y?
(c) Find the linear approximation (linearization) $L(x, y)$ of the function $R(x, y)$ about the resistances $(x, y) = (10, 40)$ Ohms.

Problem 4: (30 points)
You recently moved to the spherical planet A2P3P5M0 whose radius you have been told is $4\sqrt{3}$ units. You need to find a place to live on the surface of the planet. Thankfully, before building a residence, your realtor told you about a subterranean radiation source located at $(1, -1, 1)$. Since you do not want to glow in the dark, you want to live as far from the radiation source as possible. Where on the planet surface should you build your house? You can assume the center of the planet is at the origin of a standard xyz–coordinate system. To obtain full credit, you must use Calculus III techniques that you have learned in this course.