Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use one page of notes. You cannot collaborate on the exam or seek outside help, nor can you use the recorded lectures, a calculator, any computational software, or material you find online.

Name:

1. (26 points) Consider the two vectors:

 $\mathbf{v}_1 = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ 

 $\mathbf{v}_2 = \mathbf{i} + \mathbf{j} + \mathbf{k}$ 

- (a) (6 points) Are these vectors orthogonal? Are they parallel? Justify your answer.
- (b) (6 points) Find a nonzero vector that is orthogonal to both of these vectors.
- (c) (6 points) What is the area of the parallelogram formed by  $\mathbf{v}_1$  and  $\mathbf{v}_2$ ?.
- (d) (8 points) If we add a third vector,  $\mathbf{v}_3 = \mathbf{i} 2\mathbf{j} \mathbf{k}$ , what is the volume of the parallepiped formed by these three vectors?

2. (26 points) Consider the plane given by

$$3x - 2y + z = 2$$

and the line given by the symmetric equations

$$\frac{x+1}{2} = y = 2 - z.$$

- (a) (6 points) Find the point where the line intersects the plane.
- (b) (6 points) Find the angle between the line and plane's normal vector.
- (c) (6 points) Show that the point A(3,2,0) is on the line.
- (d) (8 points) Determine how far the point A is from the plane.

3. (20 points) A quadric surface is defined by the equation

$$2x^2 - 4x + y^2 - z^2 - 2z = 0$$

- (a) (8 points) Classify the quadric surface and state its orientation.
- (b) (6 points) Find the center of the surface.
- (c) (6 points) Sketch the trace for y = 1. Label all x and z intercepts.

4. (28 points) An ion moving in a magnetic field starts at the origin and follows a path given by the position vector

$$\mathbf{r}(t) = \left\langle \frac{t}{2} + \sin(t), \cos(t) - 1, \frac{3t}{8} \right\rangle$$

- (a) (8 points) After the ion has moved  $3\pi$  in the z direction, the ion collides with a neutral atom. Where does this collision occur? How far from the origin does the collision occur?
- (b) (8 points) Find the velocity vector and acceleration vector of the ion. What is their dot product?
- (c) (6 points) Set up, but do not evaluate, the integral to determine how far the ion has moved along it's path before the collision. Make sure to simplify the integrand and use the correct limits of integration.
- (d) (6 points) In the collision, the ion becomes a neutral atom and continues traveling in a straight line afterwards. Find an equation for this line.