1. (28 pts) The height of a hill in meters is given by

$$h(x,y) = 9 - \sqrt{3 + x^2 + y^2}, \quad x^2 + y^2 \le 78.$$

Gamma Goat is walking on the hill and has reached the point P(2,3,5).

- (a) Gamma Goat spots Beta Bee which is hovering at point Q(4, 4, 7). Find a vector equation for the line segment connecting P and Q.
- (b) If Gamma Goat decides to descend the hill as quickly as possible, in which direction should the goat walk? Express your answer in the form of a vector  $p\mathbf{i} + q\mathbf{j}$ .
- (c) Gamma Goat chooses instead to walk across the hill, maintaining its elevation above the ground. Find a vector equation for this path in the form  $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ .
- (d) The shape of the hill matches which quadric surface?
- 2. (15 pts) Zeta is building an open-top wooden rectangular box with a square base. The volume of the box will be 4000 cm<sup>3</sup>. Use Lagrange multipliers to find the box dimensions that will minimize the amount of wood needed.
- 3. (16 pts) The joint probability density function for random variables X and Y is

$$f(x,y) = \begin{cases} 3e^{-x}e^{-3y} & x > 0, \ y > 0\\ 0 & \text{otherwise.} \end{cases}$$

Compute  $P(X < Y) = \int_0^\infty \int_0^y 3e^{-x}e^{-3y} dx dy.$ 

4. (27 pts) A cylindrical silo with a spherical top has the cross-section shown at right for an arbitrary value of  $\theta$ .



Set up (but do not evaluate) integral(s) to calculate the volume of the silo

- (a) using rectangular coordinates in the order dz dy dx
- (b) using cylindrical coordinates in the order  $dz dr d\theta$
- (c) using spherical coordinates in the order  $d\rho \, d\phi \, d\theta$ .
- 5. (16 pts) Use Gaussian elimination and back substitution to solve the linear system.

$$-x + 2z = 3$$
  

$$3x + y - z = 2$$
  

$$x + y + z = -2$$

6. (17 pts) Solve the linear system by finding the inverse of the coefficient matrix.

$$\begin{array}{rcl} \frac{x}{2} & + & z = & 1\\ 2x - y & = & -3\\ x & + & 3z = & 1 \end{array}$$

7. (15 pts) Consider this linear system in variables x and y. Find nonzero constants a, b, c, and d to produce each of the following results. (There are multiple possible answers.)

$$-3x + 2y = a$$
$$3x - 2y = b$$
$$6x - cy = d$$

- (a) The system has no solutions.
- (b) The system has a unique solution.
- (c) The system has infinitely many solutions.
- 8. (16 pts) Given the points (0,2), (-1,0), (-2,1), solve a linear system to find the least-squares line of best fit.