1. (16 pts) The area of a garden is given by
$$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{9-x^2}} dy\,dx + \int_1^3 \int_0^{\sqrt{9-x^2}} dy\,dx.$$
   (a) Sketch the shape of the garden.
   (b) Use polar coordinates to combine the two integrals into one double integral. Do not evaluate the result.

2. (14 pts) A tetrahedron in the first octant is bounded by the coordinate planes and the surface $x + 2y + 3z = 6$.
   (a) Sketch and shade the projection of the region onto the yz-plane. Label the intercepts.
   (b) Set up (but do not evaluate) a triple integral to find the volume of the region using rectangular coordinates in the order $dx\,dz\,dy$.

3. (23 pts) Consider the solid with volume $V = \displaystyle\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} dz\,dy\,dx.$
   (a) Sketch a cross-section of the solid in the rz–plane (that is, a half-plane of constant θ). Label the intercepts.
   (b) Set up (but do not evaluate) an equivalent integral using
      i. cylindrical coordinates in the order $dz\,dr\,d\theta$.
      ii. spherical coordinates in the order $d\rho\,d\phi\,d\theta$.

4. (15 pts) Consider the integral
$$\int_0^1 \int_{-3x}^{x} (x-y)(3x+y)^2\,dy\,dx.$$
   (a) Sketch the region of integration in the xy-plane.
   (b) Let $u = x - y$ and $v = 3x + y$. Sketch the transformed region in the uv-plane.
   (c) Set up an equivalent integral in the uv-plane. Do not evaluate the integral.

5. (18 pts) Random variable X has the probability density function
$$f(x) = \begin{cases} \dfrac{a}{x^n} & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$
   for some constant a and $n > 1$.
   (a) Find a and express your answer in terms of n.
   (b) Set up (but do not evaluate) an integral to calculate $(X > k)$ for $k \geq 1$.

6. (14 pts) Let $B = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.
   (a) Find $B^T B$.
   (b) Let A be any matrix and let $C = A^T A$. Prove that C is symmetric.