1. (40 pts) Let $g(x, y) = x^3 - 3xy + y^3$.
   (a) Find and classify the critical points of $g(x, y)$.
   (b) Find the maximum rate of change of $g(x, y)$ at the point $(2, 1)$ and the direction in which it occurs.
   (c) The origin and the point $(2, 1, 3)$ lie on the surface $z = g(x, y)$. Find an equation for the plane that passes through the points and contains the line with symmetric equations $x = \frac{y}{3} = z$.
   (d) Starting at the origin, a fly takes off from the surface $z = g(x, y)$ and travels along the path $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + 7t^2\mathbf{k}$, $t \geq 0$. At what value(s) of t will the fly meet the surface again?

2. (15 pts) Consider the integral
$$\int_0^3 \int_{1-x}^{1+x} \frac{x-y}{x+y}\, dy\, dx.$$
Use the transformation $u = x - y$, $v = x + y$ to set up an equivalent integral over a region in the uv plane. Sketch both the xy and uv regions. Do not evaluate the integral.

3. (25 pts) The volume of a solid is given in cylindrical coordinates by $\displaystyle\int_{\pi/2}^{\pi} \int_0^6 \int_r^6 r\, dz\, dr\, d\theta$.
   (a) Sketch and shade the 2D cross-sections of the solid in the rz-plane (for a constant θ) and in the xy-plane. Label all intercepts.
   (b) Set up (but do not evaluate) an equivalent integral in rectangular coordinates in the order $dz\, dy\, dx$.
   (c) Set up (but do not evaluate) an equivalent integral in spherical coordinates in the order $d\rho\, d\phi\, d\theta$.

4. (25 pts)
   (a) Use Gaussian elimination to solve the linear system.
$$\begin{aligned} 2x + 4y &= -10 \\ x - 4y + z &= 6 \\ x + y &= -4 \end{aligned}$$
   (b) Reduce this homogeneous system to RREF and use the result to find the complete solution set.
$$\begin{aligned} 2x + 4y &= 0 \\ x - 4y + z &= 0 \end{aligned}$$

5. (15 pts) Solve the linear system by finding the inverse of the coefficient matrix.
$$\begin{aligned} 3x + 3z &= 2 \\ -x - y &= 1 \\ x + y + z &= 0 \end{aligned}$$

6. (15 pts) Consider this linear system in variables x and y. For each of the following results, find nonzero coefficients a, b, c, d, e, and f. (There are multiple possible answers.)
$$\begin{aligned} ax + by &= 10 \\ cx + dy &= 5 \\ ex + fy &= 1 \end{aligned}$$

(a) The system has no solutions.

(b) The system has infinitely many solutions.

(c) The system has a unique solution.

7. (15 pts) Find all square roots of the matrix A. Justify your answers. (Matrix B is a square root of A if $B^2 = A$.)

$$A = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$