1. (10 pts) Given vectors $\mathbf{a}$ and $\mathbf{b}$, prove that if

$$(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = |\mathbf{a}|^2 + |\mathbf{b}|^2,$$

then $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.

2. (12 pts) Evaluate $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy$ by converting to polar coordinates.

3. (28 pts) Consider the 3D region bounded by $x = y^2$, $z = 0$, and $x + 2z = 8$.

   (a) Sketch and shade the projection of the region onto the xz-plane. Label all intercepts.

   (b) Sketch and shade the projection of the region onto the yz-plane. Label all intercepts.

   (c) Set up (but do not evaluate) triple integral(s) to find the volume of the region using rectangular coordinates in the order:

      i. $dz\,dx\,dy$

      ii. $dy\,dx\,dz$

4. (20 pts) Consider the integral $\displaystyle\int_{-1}^{0}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{x^2+y^2}} dz\,dy\,dx$. Set up (but do not evaluate) equivalent integrals using

   (a) cylindrical coordinates in the order $dz\,dr\,d\theta$.

   (b) spherical coordinates in the order $d\rho\,d\phi\,d\theta$.

5. (15 pts) Consider the integral

$$\int_0^1\int_{-2y}^{2y}(x-2y)\sqrt{x+2y}\,dx\,dy.$$

   Use the transformation $u = x - 2y$, $v = x + 2y$ to set up an equivalent integral over a region in the uv-plane. Sketch both the xy and uv regions. Do not evaluate the integral.

6. (15 pts) Libra and Leo decide to meet at Fiske Planetarium to see a show. Suppose each independently arrives at a time uniformly distributed between 6:40 and 7:00 pm. Evaluate a double integral to find the probability that the first to arrive has to wait longer than 8 minutes.