1. (31 pts) Let $g(x,y) = 2x^2 + 3y^2$.
   (a) i. Find a linear approximation for $g(x,y)$ centered at $(1,-1)$. You may leave your answer unsimplified.
   ii. Use the linear approximation to estimate the value of $g(1.2,-1.2)$.
   iii. Find an error bound for the estimate calculated in part ii.
   (b) Find an equation for the plane tangent to the surface $z = g(x,y)$ at $x = 1, y = -1$. Simplify your answer.
   (c) Let $h(x,y) = \dfrac{xy}{g(x,y)}$. Show that $\lim\limits_{(x,y)\to(0,0)} h(x,y)$ does not exist.

2. (25 pts) You are hiking on Turtle Mountain. The elevation on the mountain is given by

$$z = 1000 - \frac{1}{200}x^2 - \frac{1}{100}y^2$$

   where x, y, and z are measured in meters. Suppose the positive x-axis points east and the positive y-axis points north. When you reach the location $x = 60$, $y = 40$, you stop to consider four options before continuing the hike.
   (a) Option 1: You hike due south (i.e., in the negative y direction). Will you start to ascend or descend? At what rate?
   (b) Option 2: You hike in the northwest direction. Will you start to ascend or descend? At what rate?
   (c) Option 3: You maintain the same elevation as you hike. In which direction should you head? Write your answer as a unit vector.
   (d) Option 4: You go home, taking the shortest route to the base of the mountain where you will catch a shuttle. In which direction should you head? What is the initial rate of descent?

3. (20 pts) Nemo and Dory wish to build a backyard pool for their prized collection of fish. The rectangular pool will hold 2000 cubic feet of water. The sides of the pool, to be lined with decorative tile, will cost twice as much per square foot as the concrete bottom. Use Lagrange multipliers to find the pool dimensions that will minimize the cost of construction.

4. The following two problems are not related.
   (a) (12 pts) Find and classify all critical points of $f(x,y) = x^2e^y + ye^y$.
   (b) (12 pts) The temperature in degrees Celsius at a point (x,y,z) is given by

$$T(x,y,z) = 2x^2 - xyz.$$

   A particle moving through space has the position function $\mathbf{r}(t) = 2t^2\mathbf{i} + 2t\mathbf{j} - t^2\mathbf{k}$, where time t is measured in seconds. In degrees per second, how fast is the temperature along the particle's path changing when it reaches the point $\left(\frac{1}{2}, 1, -\frac{1}{4}\right)$?