**Work out the following problems, fully simplifying your answers.**

1. (30 pts) Evaluate the following integrals.

$$(a)\ \int_0^{\pi/2} \sin^2(\theta)\cos^3(\theta)\,\mathrm{d}\theta \qquad (b)\ \int_2^{\infty} \frac{2}{(1+x)(1-x)}\,\mathrm{d}x$$

2. (15 pts) Solve the following initial value problem for $y$ as a function of $x$.

$$\begin{cases} \frac{\mathrm{d}y}{\mathrm{d}x} = -2xy \\ y(0) = 2 \end{cases}$$

3. (20 pts) Determine the radius and interval of convergence for the following power series.

$$\sum_{n=0}^{\infty} \frac{10^n (x-5)^n}{n!}$$

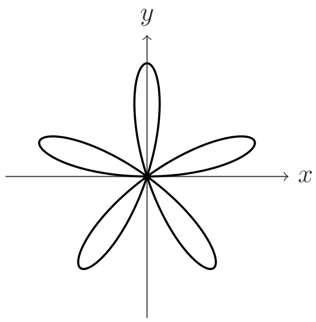
4. (25 pts) Given the function $f(x) = \sin(x)\cos(x)$, answer the following:
   (a) Using any method you'd like, compute $T_3(x)$ for the Maclaurin series of $f(x) = \sin(x)\cos(x)$.
   (b) Assuming $\left|f^{(4)}(x)\right| \le 8$ for all values of $x$, find an error bound in using $T_3(x)$ to approximate $f(-0.1)$.

5. (30 pts) Consider the parametric equations given below.

$$\begin{cases} x = t^2 \\ y = \sin t \end{cases}, \qquad 0 \le t \le \pi$$

   Answer the following:
   (a) Setup and evaluate an integral with respect to $t$ to find the area between the curve and the $x$-axis.
   (b) Assuming $t \ge 0$, eliminate the variable $t$ from the parametric equations to find an equation of the curve in terms of $x$ and $y$.

6. (30 pts) Consider the polar curve defined by $r = \sin(5\theta)$ for $0 \le \theta \le \pi$ (plotted below).




   Answer the following:
   (a) Setup, **but do not evaluate**, an integral to find the total length of the curve $r = \sin(5\theta)$.
   (b) Evaluate an integral to the find the area enclosed by one petal of the curve $r = \sin(5\theta)$.

FORMULAS ON BACK

**Trigonometric Identities**

$$\cos^2 x = \frac{1}{2}(1+\cos 2x) \qquad \sin^2 x = \frac{1}{2}(1-\cos 2x) \qquad \sin 2x = 2\sin x\cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

**Common Maclaurin Series**

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$