## Work out the following problems and simplify your answers.

1. $(30 \mathrm{pts})$ Evaluate the following integrals.
(a) $\int t \sin (3 t) \mathrm{d} t$
(b) $\int_{-1}^{1} \frac{1}{x} \mathrm{~d} x$
2. $(20 \mathrm{pts})$ Consider the sequence $\left\{a_{n}\right\}_{n=1}^{\infty}$ where $a_{n}=\frac{n+1}{n}-\frac{n+2}{n+1}$.
(a) Does the sequence $\left\{a_{n}\right\}$ converge? If so, find its limit. If not, explain why not.
(b) Using the sequence $\left\{a_{n}\right\}$ given in the problem, does $\sum_{n=1}^{\infty} a_{n}$ converge? If so, find its sum. If not, explain why not.
3. (20 pts) The Maclaurin series for $\operatorname{sinc} x=\frac{\sin x}{x}$ is $\sum_{n=0}^{\infty}(-1)^{n} \frac{x^{2 n}}{(2 n+1)!}$.
(a) Find the radius of convergence of the series.
(b) Using the series, what is the value of $\operatorname{sinc}(0)$ ?
(c) With the series above in mind, compute the sum of $\sum_{n=0}^{\infty}(-1)^{n} \frac{\pi^{2 n}}{(2 n+1)!6^{2 n}}$.
4. (25 pts) The following problems are related.
(a) Find the 3 rd degree Taylor polynomial $T_{3}(x)$ centered at $a=1$ of $\ln x$.
(b) Estimate the error in using $T_{3}$ to approximate $\ln x$ at $x=\frac{3}{2}$.
5. (25 pts) Suppose the trajectory of a projectile launched from a cannon is given by the parametric curve

$$
x=10-10 e^{-t}, y=11-11 e^{-t}-t, \quad t \geq 0
$$

where $t$ is the time from launch. Setup, but do not evaluate, integrals to find the following:
(a) The distance the projectile has traveled from $t=0$ to $t=10$.
(b) The area between the trajectory and the $x$-axis from $t=1$ to $t=5$.
6. (30 pts) Consider the two polar equations $r=4 \cos \theta$ and $r=2$. Answer the following:
(a) Sketch both polar curves and label their intersections.
(b) Find the area of the region inside of $r=4 \cos \theta$ and outside of $r=2$.

