## Work out the following problems and simplify your answers.

1. (30 pts) Evaluate the following integrals.

(a) 
$$\int t \sin(3t) dt$$
 (b)  $\int_{-1}^{1} \frac{1}{x} dx$ 

2. (20 pts) Consider the sequence 
$$\{a_n\}_{n=1}^{\infty}$$
 where  $a_n = \frac{n+1}{n} - \frac{n+2}{n+1}$ 

- (a) Does the sequence  $\{a_n\}$  converge? If so, find its limit. If not, explain why not.
- (b) Using the sequence  $\{a_n\}$  given in the problem, does  $\sum_{n=1}^{\infty} a_n$  converge? If so, find its sum. If not, explain why not.

3. (20 pts) The Maclaurin series for sinc 
$$x = \frac{\sin x}{x}$$
 is  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$ 

- (a) Find the radius of convergence of the series.
- (b) Using the series, what is the value of sinc(0)?
- (c) With the series above in mind, compute the sum of  $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n+1)! 6^{2n}}$ .
- 4. (25 pts) The following problems are related.
  - (a) Find the 3rd degree Taylor polynomial  $T_3(x)$  centered at a = 1 of  $\ln x$ .
  - (b) Estimate the error in using  $T_3$  to approximate  $\ln x$  at  $x = \frac{3}{2}$ .
- 5. (25 pts) Suppose the trajectory of a projectile launched from a cannon is given by the parametric curve

 $x = 10 - 10e^{-t}, \ y = 11 - 11e^{-t} - t, \qquad t \ge 0$ 

where t is the time from launch. Setup, but do not evaluate, integrals to find the following:

- (a) The distance the projectile has traveled from t = 0 to t = 10.
- (b) The area between the trajectory and the x-axis from t = 1 to t = 5.
- 6. (30 pts) Consider the two polar equations  $r = 4 \cos \theta$  and r = 2. Answer the following:
  - (a) Sketch both polar curves and label their intersections.
  - (b) Find the area of the region inside of  $r = 4 \cos \theta$  and outside of r = 2.