Work out the following problems and simplify your answers.

1. (30 pts) Determine if the following series converge or diverge. Fully justify your answer and state which test you used.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}n!}{(2n)!}$

2. (15 pts) Find the sum of the following series $\displaystyle\sum_{n=0}^{\infty} \frac{3}{n^2+3n+2}$. (Hint: use partial fractions)

3. (20 pts) Consider the series $\dfrac{1}{10} - \dfrac{1}{20} + \dfrac{1}{30} - \dfrac{1}{40} \pm \cdots$.

   (a) Find a_n and write the series in sigma notation, $\displaystyle\sum_{n=1}^{\infty} a_n$.

   (b) Is the series absolutely convergent, conditionally convergent, or divergent. Justify your answer.

   (c) How many terms are needed to estimate the actual sum to an error less than 10^{-3}?

4. (15 pts) Suppose $f(x)$ has a power series representation $\sum_{n=0}^{\infty} c_n(x-a)^n$ with an interval of convergence of $(0, 2]$.

   (a) Find the center and radius of convergence of the series.

   (b) Determine whether the following series are convergent, divergent, or if more information is needed to make a conclusion. Justify your answers.

   (i) $\displaystyle\sum_{n=0}^{\infty} c_n \frac{1}{2^{n+1}}$ (ii) $\displaystyle\sum_{n=0}^{\infty} c_n 2^{3n}$ (iii) $\displaystyle\sum_{n=0}^{\infty} c_n(-1)^n$

5. (20 pts) Compute the following stating the radius of convergence for each part.

   (a) Write out the power series centered at 0 for $\dfrac{1}{1-x}$.

   (b) Find a power series centered at 0 of $\dfrac{1}{3+x}$.

   (c) By integrating, find a power series for $\ln(3+x)$.

   (d) Find the power series for $x^2 \ln(3+x)$.