Answer the following problems and simplify your answers.

1. (18pts) Find the **explicit solution** to the following initial value problem:

$$\begin{cases} \dfrac{\mathrm{d}z}{\mathrm{d}t} - e^{t+z} = 0 \\ z(0) = \ln 2 \end{cases}$$

2. (18 pts) Conisder the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ on the interval $\dfrac{1}{2} \le x \le 1$.

   (a) Find the area of the surface obtained by rotating the curve about the y-axis.

   (b) Set up, **but do not evaluate**, the integral with respect to x to find the area of the surface rotated about $y = -2$.

3. (40 pts) Consider the region $\mathcal{R}$ bounded by $y = \frac{1}{2}x^2$ and $y = \sqrt{2x}$.

   (a) Sketch and shade $\mathcal{R}$, labeling the axes, intersections points, and curves.

   (b) Set up, **but do not evaluate**, integrals to find the following quanities **with respect to dx**:

       i. The volume of the solid generated by rotating $\mathcal{R}$ about $x = -1$.

       ii. The volume of the solid generated by rotating $\mathcal{R}$ about $y = 2$.

       iii. The volume with a base of $\mathcal{R}$ and rectangular cross-sections perpendicular to the x-axis that have a height 3 times the length of their base.

   (c) Assuming a uniform density ρ, find the y-coordinate of the centroid of $\mathcal{R}$. **Fully simplify your answer.**

4. (24 pts) Determine whether or not the following sequences converge or diverge. Justify your answer! If the sequence converges, find its limit.

   (a) $\left\{ \dfrac{(-1)^{n+1}n}{n^{3/2} + \sqrt{n}} \right\}$ (b) $\left\{ \ln(2n^2 + 1) - 2\ln(n+1) \right\}$ (c) $\left\{ 1 + 4^n \cdot 3^{2-n} \right\}$